

2500
8
200 MAX.

SHREWSBURY SCHOOL

MATHEMATICS PRIZE, 1957

1. (i) Evaluate $\sqrt[3]{\log(0.7)}$ correct to three significant figures. M
10 (5)
 (ii) The number N is equal to 2^{100} . How many digits does N contain and what are the first two digits? 15 (8)

X 2. The difference between the simple and the compound interest on a certain sum for 3 years at 5% is £6.3s. Find the sum. 25 (15)

3. An endurance test for racing cars takes place on a track which consists of a closed circuit of length 14 miles. Three cars, having speeds of 150, 143 and 129 feet per second respectively, set off from the starting point at the same instant and travel in the same direction round the course. Find the distance, measured in miles along the track from the starting point, of the place at which all three cars are first alongside each other again. 25 (15)

4. Factorize the following :
 (i) $x^3 - 729x$ (ii) $x^2 - y^2 + 3z^2 + 2yz + 4zx$ (iii) $p^4 - 12p^2 + 4$ 4p² - 16p²
12 (7)

5. Solve the equations : (i) $9^x - 2(3^x) = 3(3^{x+1} - 8)$ 144 Eqn in 3^x (8)
Answer 2, 4
 (ii) $\frac{2}{x+8} + \frac{5}{x+9} = \frac{3}{x+15} + \frac{4}{x+6}$ 11 Simplify (put in 5)
(Answer 6)

6. ABC is a triangle in which $AB = AC$. The line bisecting the angle B meets AC at D. K is the point which is equidistant from B and D and such that KA is parallel to BC. 25

Prove that a circle can be drawn through the points B, D, A, K.

7. ABC is any triangle. K is the mid-point of BC and L is the point on AC which is between A and C and such that $AL : LC = 2 : 1$. AK and BL intersect at X. 25

Prove (i) $AX : XK = 4 : 1$, (ii) $BX : XL = 3 : 2$.

Further, if CX produced meets AB at M, find the ratio in which M divides AB.

8. Prove that the sum of the diagonals of a quadrilateral is greater than the semiperimeter and less than the perimeter of the quadrilateral. 13
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