

SHREWSBURY SCHOOL

MATHEMATICS PRIZE, 1964

1. (i) Find the sum of all the numbers between 0 and 400 which are multiples of 7 and 13.

(ii) How many numbers between 0 and 100 have squares whose last digit is 5 or 6?
2. A number of cubes, of identical size, have each of their faces either black or white. How many can there be if all the cubes are different?
3. It is required to find two numbers, each of two digits, such that the first number is equal to the product of the digits of the second number, and is also less by 100 than twice the second number. Find all positive solutions.
4. Solve for x , y , z and w the simultaneous equations:
$$\begin{aligned}x + y + z &= a, \\x + y + w &= b, \\x + z + w &= c, \\y + z + w &= d.\end{aligned}$$
5. Given a circle and a point A inside it, show how to construct a chord PAQ so that $PA = 2 AQ$.
6. A man wishes, by paying a cash price, to purchase an annuity of 10 payments of £100 at yearly intervals, the first being one year after his purchase. Assuming him to be entitled throughout to compound interest at 3% per annum on all money not paid over to him, what is the purchase price?
7. In an examination 127 candidates sat for all three subjects Physics, Chemistry and Biology. There were 82 passes in Physics, 91 in Chemistry and 102 in Biology. 66 candidates passed both Physics and Chemistry, 74 passed both Chemistry and Biology, and 70 both Physics and Biology. There were 57 candidates who passed in all three subjects. How many failed all?

[P.T.O.]

8. In the clan McBine, each man has two sons who have different names. The only Christian names given in the clan are Duncan, Ian and Keith. No son is named after his father. If S^P is the number of a man's (great) P -grandsons who bear his Christian name, show that

$$S_p = 2^P + S_p - 2$$

9. The "Russian" method of multiplication, of which an example is given below, is as follows: Of the two numbers to be multiplied one is divided successively by 2 until unity is reached, while the other is correspondingly multiplied by 2. In the divisions, remainders are ignored. Even numbers in the division sequence, together with the corresponding entries in the multiplication sequence, are struck out. The remaining numbers in the multiplication sequence are added to give the desired product.

Justify the process. Your proof may refer to the example below. Verification by a decimal multiplication will not be acceptable.

Example:— Multiply 167 by 458.

167	458	Strike	167	458
83	916	out	83	916
41	1832	"evens"	41	1832
20	3664	
10	7328	
5	14656		5	14656
2	29312	
1	58624		1	58624
				76486 = 167 × 458