

Attempt as many questions as possible. More difficult questions (with a greater number of marks) occur towards the end.

1. In figure 1, where should the plank be placed in order to minimise the journey from A to B?

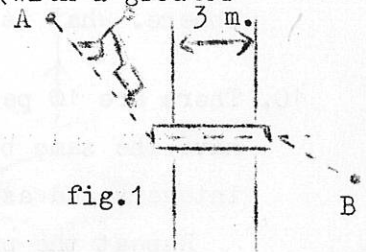


fig.1

2. A snooker ball at P is to be pocketed at Q (see figure 2). Because of obstructions it is necessary to aim via AB and BC. Assuming that the ball bounces off a cushion at the same angle as it strikes it, what is the shortest path from P to Q?

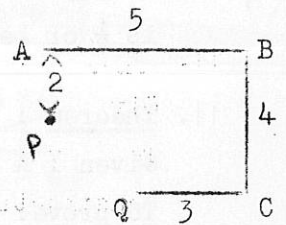


fig.2

3. A signaller has six flags, one red, two yellow, and three green. How many different messages can he send using any five of the flags?
4. A commuter is in the habit of arriving at his suburban station each evening exactly at five o'clock. His wife always meets the train and drives him home. One day he takes an earlier train, arriving at the station at four. The weather is pleasant, so instead of telephoning home he starts walking along the route always taken by his wife. They meet somewhere on the way. He gets into the car and they drive home, arriving at the house ten minutes earlier than usual. Assuming that the wife always drives at constant speed, and that on this occasion she left just in time to catch the five o'clock train, can you determine how long the husband walked before he was picked up?

5. Assign one of the following dates to each part of the question. Each should be used once and only once.

third millennium B.C., sixth century B.C., 330 B.C., third century B.C., 1545, 1614, 1637, 1687, 1813, 1905

- (i) Treatise of Nicholas Copernicus 'Concerning the Revolution of Heavenly Bodies' published.
- (ii) Isaac Newton's first published work on 'the calculus'.
- (iii) When did Pythagoras live?
- (iv) Einstein developed his theory of special relativity.
- (v) René Descartes' first description of a point by means of its distance from two axes.
- (vi) Babylonian development of place value notation.
- (vii) John Napier first published his work on logarithms.
- (viii) Argand first published his work on complex numbers in?
- (ix) Archimedes lived in?
- (x) Euclid published his treatise entitled 'Elements' in?

6. Show that $2^n + 1$ or $2^n - 1$ is divisible by 3, according as n is odd or even.

7. Sketch the curve $y^2 x^2 = (4 - y^2)(y - 1)^2$. Explain why you think the graph takes the form you have drawn.

8. Solve the simultaneous equations :

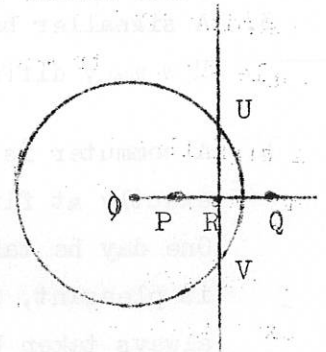
$$x^3 + y^3 = 8, \quad xy + (x + y) = 2.$$

(Hint : put $x + y = u$, $xy = v$. Note that $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$)

9. A cylindrical hole 10 cm long has been drilled through the centre of a solid sphere. What is the volume of the sphere remaining?
10. There are 10 people in a room. What is the probability that at least two of them have the same birthday (i.e. day and month). Leave your answer as a product of integers and assume a 365 day year.
- Repeat the problem for n people. Suggest a value for n so that the probability is $\frac{1}{2}$ or better.

11. Theorem : To prove that every point inside a circle lies on its circumference.
 Given : A circle, centre O , radius r , and an arbitrary point P inside it.
 To prove: That P lies on the circumference.

Construction : Let Q be the point on OP produced beyond P such that $OP \cdot OQ = r^2$, and let the perpendicular bisector of PQ cut the circle at U, V . Denote by R the mid-point of PQ .



Proof : $OP = OR - RP$

$$OQ = OR + RQ = OR + RP \quad (RQ = RP, \text{ construction})$$

$$\therefore OP \cdot OQ = (OR - RP)(OR + RP) = OR^2 - RP^2$$

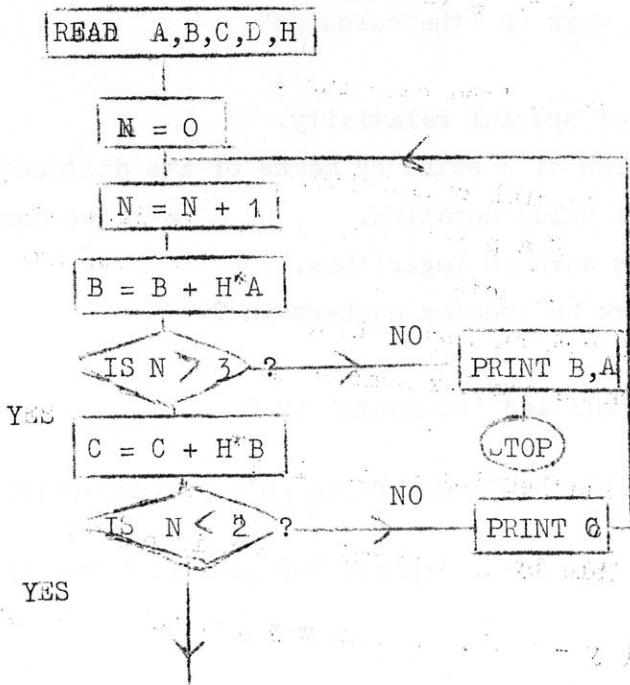
$$= (OU^2 - RV^2) - (PU^2 - PV^2) \quad (\text{Pythagoras})$$

$$= OU^2 - PU^2 = OP \cdot OQ - PU^2 \quad (OP \cdot OQ = r^2 = OU^2)$$

$\therefore PU = 0 \quad \therefore P$ is at U , on the circumference. EXPLAIN.

12. Express $x^3 - 3x^2 + x + 2$ in the form $p(x - 2)^3 + q(x - 2)^2 + r(x - 2) + s$, where p, q, r, s are constants to be found.

Given a cubic polynomial $P(x) = Ax^3 + Bx^2 + Cx + D$, the following (incomplete) flow diagram is intended (by dividing $P(x)$ repeatedly by $x - H$) to find the constants appropriate for expressing $P(x)$ in terms of $x - H$. It contains an error.



* denotes multiplication

- (i) Briefly describe the purpose of N .
- (ii) Correct the error and complete the program.