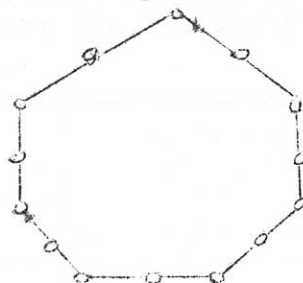


MATHEMATICS PRIZE 1979

Attempt as many questions as you like. The questions do not necessarily carry equal marks. SHOW ALL WORKING, AND EXPLAIN WHAT YOU ARE DOING. Answers to full questions will receive greater credit than fragmentary solutions.

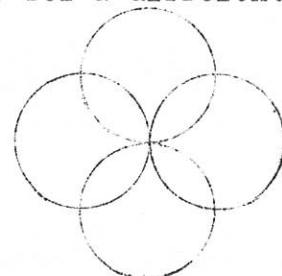
1. a) Show that if the sum of the digits of the number $a_0b_0c_0d_0$ is divisible by 11, then the number itself is also divisible by 11.
 - b) A number when divided by 2 gives remainder 1, when divided by 3 gives remainder 2, when divided by 4 gives remainder 3, when divided by 5 gives remainder 4, when divided by 6 gives remainder 5, when divided by 7 gives remainder 6, when divided by 8 gives remainder 7, when divided by 9 gives remainder 8, when divided by 10 gives remainder 9, and is less than 3,000. What is the number?
2. a) What is the shortest distance an ant will travel if he walks from one vertex of a cube of side 1 m. to the opposite one, along the surfaces of the cube?
 - b) If $x^2 = 2x - 3$, show that $x^5 = -11x + 12$.

3. Using each of the numbers 1, 2, 3, ..., 14 once only, put a number in each of the circles such that the sum of the numbers on each of the seven straight lines is 19.



4. In the following division problem, each letter stands for a different digit. Find what each letter stands for.

$$\begin{array}{r}
 \text{PR)MTVVR(RSR)} \\
 \underline{\text{MVR}} \\
 \text{KKV} \\
 \underline{\text{KMD}} \\
 \text{MVR} \\
 \underline{\text{MVR}}
 \end{array}$$



5. a) Four cyclists, starting at noon at the point of intersection of the four circles, each of circumference $\frac{1}{3}$ mile, ride round a different circle. One rides at 6 m.p.h., another at 9 m.p.h., another at 12 m.p.h., another at 15 m.p.h.. They agree to ride until they meet at the centre for the fourth time. When did they finish? (see above diagram)
- b) Express the decimal 2.713713713..... as a fraction in its lowest terms.
6. a) If two ~~queens~~ ^{castles} are placed at random on a chessboard, what is the probability that they can take each other?
- b) Repeat part a) for two knights
7. Three men and their wives hire a van with 6 seats: one beside the driver, two with their backs to the driver, and two behind, facing the driver. If only the men can drive, how many possible arrangements for seating are there if
- no two women sit next to each other?
 - if the three men all sit on the right hand side?
 - if the pair sitting at the front are husband and wife?
8. Given that $x^2 + mx + 1 = (x - a)(x - b)$, and $x^2 + nx + 1 = (x - a^3)(x - d)$, show that
- $d = b^3$
 - $n = m^3$
 - $(x - a^9)(x - b^9) = x^2 + (n^3 - 3n)x + 1$

9. Adam, Ben, Celia, David and Edward each apply to two of the following five Cambridge Colleges: Clare, King's, Peterhouse, St. John's and Trinity. No two candidates select the same College as their first choice, nor have any two selected the same College as their second choice. One candidate is reading each of the following subjects: Classics, English, History, Law, Modern Languages. King's and Clare accept both men and women, but other Colleges take only men. Peterhouse does not accept Modern Linguists. Given that: i) Celia's second choice is the Lawyer's first choice; ii) neither of David's choices has been chosen by the Historian; iii) Edward is not going to read English or Law; iv) The Historian's second choice is Adam's first choice; v) Edward's second choice is Clare; vi) Adam is a Modern Linguist; discover what subject David is reading, and what are his first and second choice Colleges.

10. a) Show that the equation of the family of circles which passes through the points $(2,0)$ and $(1,0)$ is $x^2 + y^2 - 3x - 2ay + 2 = 0$, where a is the y -coordinate of the centre. (The equation of a circle centre (p,q) and radius r is $(x - p)^2 + (y - q)^2 = r^2$).
- b) Find the equation to the locus of a point P which moves so that its distances from the points $A(3,2)$ and $B(1,-5)$ are in the ratio $3:2$.

CMBW