ARNOLD HAGGER MATHEMATICS PRIZE

WEDNESDAY 11 FEBRUARY 1998, 7.15 P.M.

ONE AND A HALF HOURS

There is a total of 90 marks available for this paper. Answer as many questions as you can. Do not necessarily expect to finish the paper.

In marking the scripts, we will be looking for ELEGANT solutions. It is important that you show your reasoning clearly, and do not just write down answers.

You may use calculators in any question.

1. After a cyclist has gone two thirds of his journey he gets a puncture. Finishing on foot he spends twice as long walking as he did riding.
   How many times as fast does he ride as walk?

   (5 marks)

2. On the tear-off sheet is a circle. Using only ruler and compasses, find the centre of the circle. You should make sure all your construction marks are clear.

   (5 marks)

3. An artist has two children, the elder of which is called Maria. What is the probability that both children are girls?
   A musician has two children, one of which is called Maria. What is the probability that both children are girls?

   (5 marks)

4. Prove that the product of 4 consecutive positive integers cannot be a perfect square.

   (7 marks)

5. I decide to put a boy in Extra Lesson and ask him to write out all the rearrangements of BRANDY. (e.g. BRANYD or RBANDY - all 6 letters must be used, but the arrangement does not need to be a meaningful word.)
   By making relevant calculations and estimations, explain whether this is a fair punishment.

   (7 marks)

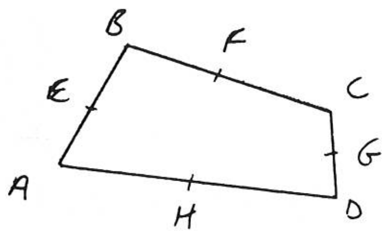
6. Evaluate $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots\ldots}}}}$

   (7 marks)

7. There is a five digit number N. With a 1 after it, it is three times as large as with a 1 before it. What is N and what is interesting about it?

   (7 marks)

8. The diagram shows a quadrilateral ABCD, with the mid-points of each side marked. Prove that EFGH is a parallelogram.

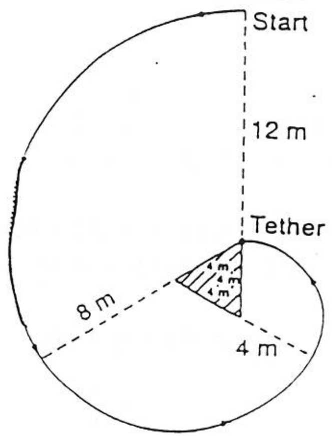


(7 marks)

9. You are told that a, b, and c are all real numbers greater than 1, and that $a^{(b^c)} = (a^b)^c$.
Find one set of values of a, b, and c satisfying this relationship where a, b, and c are all integers.
If $c = 4$, find values of a and b which satisfy the relationship.

(10 marks)

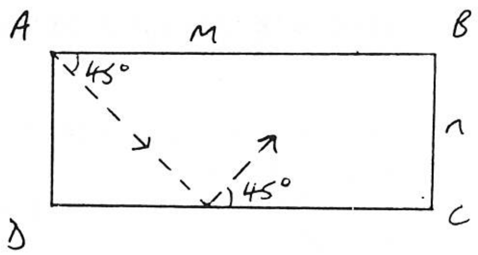
10. A goat is tethered to one corner of a pillar with an equilateral triangle as cross-section, of side 4 metres. The rope is 12 metres long and initially follows the line of one of the sides of the pillar adjacent to the tether (as illustrated). The goat (wondering why it was there in the first place) walks once around the pillar in the direction shown, keeping the rope always taut. The goat's path is shown.



How far does the goat walk?
How far would the goat walk if the pillar's cross-section was a square of side 3 metres.
How far would the goat walk if the pillar had a circular cross-section of circumference 12 metres?

(10 marks)

11. The diagram shows an adapted snooker table which has length m and width n where m and n can only take integer values. The table only has pockets at the four corners A, B, C, and D.



The ball is started from point A and is directed at 45° to the side AB. When the ball hits any cushion it rebounds symmetrically at 45° as shown in the diagram. Assume that the table is smooth and so the ball continues until it enters one of the pockets. (Ignore any complicated effects of the ball hitting the jaws of the pockets.)

If the ball enters pocket C what can you say about m and n ?
If the ball enters pocket B what can you say about m and n ?

(10 marks)

12. A new way of measuring the distance between two points is devised. It is simply the sum of the difference in x co-ordinates and the difference in y co-ordinates.

e.g. the distance from (1,3) to (5,2) is 4+1 = 5 units
the distance from (2,-1) to (2, 5) is 0+6 = 6 units

For each of the following, sketch the set of points on a diagram.

i) Find the set of points a distance of 4 units from the origin.
ii) Find the set of points equidistant from (0,0) and (2,2).

(10 marks)

USE THIS SHEET FOR YOUR ANSWER TO QUESTION 2

Tear off the sheet and hand it in with your other answers