



SHREWSBURY SCHOOL
MATHEMATICS DEPARTMENT

ARNOLD HAGGER PRIZE

24 January 2001

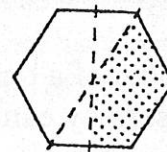
Time: $1\frac{1}{2}$ hours

Calculators are not to be used.

Attempt as many questions as possible. Marks for each question are shown in brackets. Your solutions must show full mathematical reasoning.

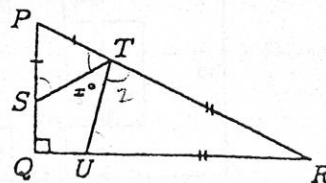
1. Two fractions are equally spaced between $\frac{1}{4}$ and $\frac{2}{3}$. What is the smaller of the two unknown fractions? (3)

2. Each of the dashed lines drawn on this regular hexagon is an axis of symmetry. What fraction of the area of the hexagon is shaded? (3)



3. A normal duck has two legs. A lame duck has one leg. A sitting duck has no legs. There are 33 ducks with a total of 32 legs. The total number of normal ducks and lame ducks is twice the number of sitting ducks. How many lame ducks are there? (4)

4. Triangle PQR is right angled at Q and triangles PST and RTU are isosceles as shown. If $\angle STU$ measures x° , what is the value of x ? (4)



TURN OVER

5. The diagram shows a 5 by 5 table. The blank squares, including the shaded one, can be filled with P's, Q's, R's, S's and T's such that no row, column or diagonal contains the same symbol more than once. What symbol must be placed in the shaded square?

P	Q	R	S	T
	P	Q	R	

(4)

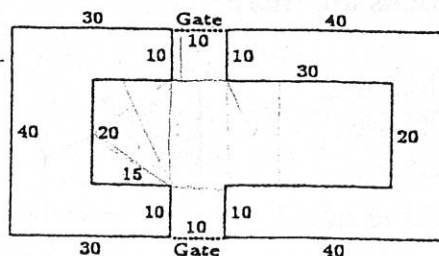
6. Two numbers x and y satisfy three of the following equations but do not satisfy the remaining one. What is the value of x ?

$$x + y = 63 \quad x - y = 47 \quad xy = 392 \quad x/y = 8 \quad (4)$$

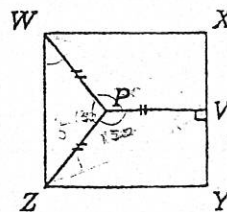
7. A floor tile has the shape of a regular polygon. If the tile is removed from the floor and rotated through 50° , it will fit back exactly into its original place on the floor. What is the least number of sides that the polygon can have? (5)

8. In Sydney Australia, the ferry takes 33 minutes to travel from Manly Wharf to Circular Quay while the hydrofoil takes 15 minutes to cover the same journey. If, on a certain day, the ferry leaves Manly for Circular Quay at 11.53 am and the hydrofoil departs at 12.05 pm, at what time will the hydrofoil overtake the ferry? (5)

9. Here is a plan of a building which has a courtyard with two entrance gates. Passers-by can look through the gates but may not enter. Dimensions of the building are given in metres, and all corners are right angles. What is the area, in square metres, of that part of the courtyard which **cannot be seen** by passers-by? (5)



10. WXYZ is a square with PV perpendicular to XY. If $PW = PZ = PV = 10$ cm, what is the area of WXYZ in square centimetres?

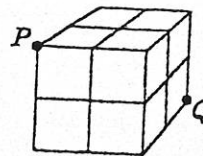


(5)

11. On my car, a particular brand of tyre lasts 40000 kilometres on a front wheel or 60000 kilometres on a rear wheel. By interchanging the front and rear tyres, what is the greatest distance, in kilometres, I can get from a set of four tyres?

(5)

12. Each face of a solid cube is divided into four as indicated on the diagram. Starting from vertex P, paths can be travelled to vertex Q along connected line segments. If each movement along the path takes one closer to Q, how many possible paths are there from P to Q?



(5)

13. In a trapezium PQRS, PQ is parallel to SR; $\angle SPQ = \angle RQP = 135^\circ$. The trapezium contains an inscribed circle and the length of PQ is 1 cm. What is the length of QR?

(8)

14. The numbers p, q, r, s and t are consecutive positive integers, arranged in increasing order. If $p + q + r + s + t$ is a perfect cube and $q + r + s$ is a perfect square, what is the smallest possible value of r?

(8)

15. How many distinct solutions consisting of positive integers has the following system of linear equations? (Note that a solution of the system is a set of values $x_1, x_2, \dots, z_2, z_3$ which satisfies all equations simultaneously).

$$\begin{array}{rcccccl}
 x_1 + x_2 + x_3 & & & & = & 5 \\
 & y_1 + y_2 + y_3 & & & = & 5 \\
 & & z_1 + z_2 + z_3 & & = & 5 \\
 x_1 & + y_1 & + z_1 & & = & 5 \\
 x_2 & + y_2 & + z_2 & & = & 5 \\
 x_3 & + y_3 & + z_3 & & = & 5
 \end{array}$$

(10)

THE END