

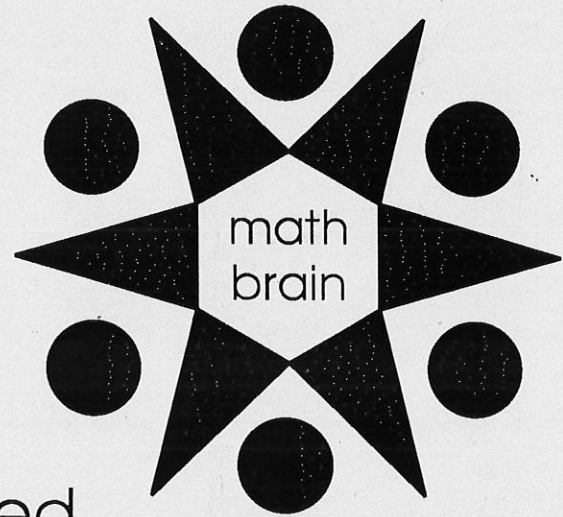


2003

THE
ARNOLD HAGGER
MATHEMATICS PRIZE
COMPETITION

WEDNESDAY 5th February
7.15pm - 8.45pm
Room M7

Calculators may NOT be used



3.1415926535897932384626433832795028841971693937

51058209749445923078164062862089986280348

253421170679821480865132823066470938446095505822317253594081284811174502

The Arnold Hagger Mathematics Prize Competition 2003

- * Questions may be answered in any order.
- * Make your methods of solution clear by including all working and reasoning.
- * The marks allocated to each question is shown - either [5], [10], [15] or [20] marks.
- * Calculators may NOT be used.

Question One : Q·U·A·C·K·E·R·S

- [5] A normal duck has two legs.
 A lame duck has one leg.
 A sitting duck has no legs.
 There are 66 ducks with a total of 64 legs.
 The total number of normal ducks and lame
 ducks is twice the number of sitting ducks.
 How many lame ducks are there ?

Question Two : S·A·F·E·T·Y F·I·R·S·T

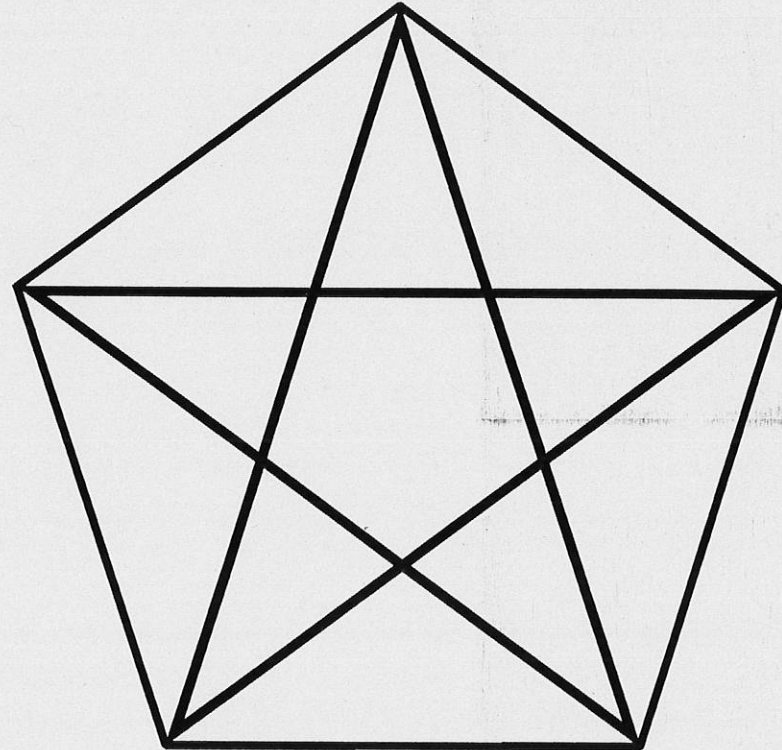
- [5] C R O S S To the left is an addition sum in disguise.
 + R O A D S Each letter stands for a different digit.
 ----- S, for example, represents 3.
 D A N G E R What do the other letters represent ?

Question Three : 1·0·1

- [5] How many positive integers less than 101 are
 NOT divisible by 3 or by 5 ?

Question Four : T·R·I, T·R·I A·N·D T·R·I A·G·A·I·N.

- [5] How many triangles are there in the following figure ?



Question Five : M·I·N·D E·S·C·A·P·E

[5] You are shipwrecked on an island inhabited by two tribes; Knights (who always tell the truth), and Knaves (who always lie). When captured you are looked after by two friendly jailers who are called Tweedledum and Tweedledee.

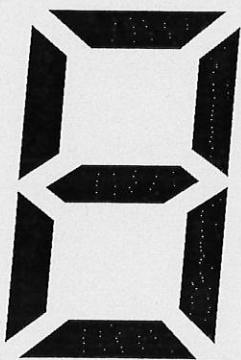
Island law states that you can regain your freedom if you discover which tribes your two jailers belong.

To help you, Tweedledum says,
"We come from different tribes".
But Tweedledee immediately says
"Oh no we don't !".

What can you conclude about your two jailers ?



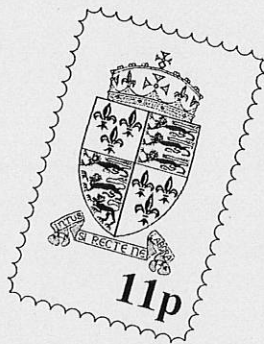
knighTs
Always ~~TELL~~ tHe
TRUTH



32

Question Six : A G·R·E·A·T E·I·G·H·T

[5] What is the units digit of 8^{32} ?



Question Seven : S·T·A·M·P·I·N·G O·N I·N·F·I·N·I·T·Y

[15] I have lots of 11p and 17p stamps.

What is the largest amount I cannot make up using just these two types of stamp ?



Question Eight : I P·R·E·F·E·R P·I

[5] A number is said to be palindromic if it reads the same backwards as forwards.
 For example, the number 7891987 is palindromic.
 Show that all 4-digit palindromic numbers are divisible by 11.

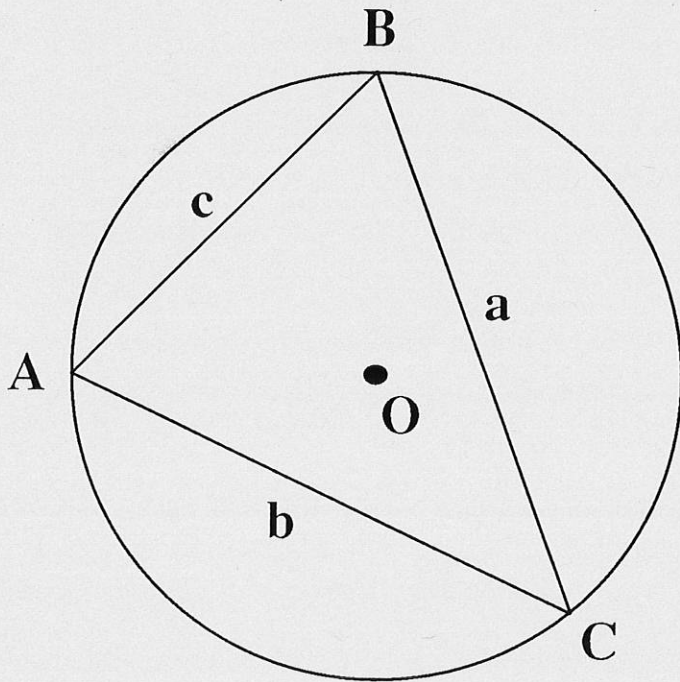
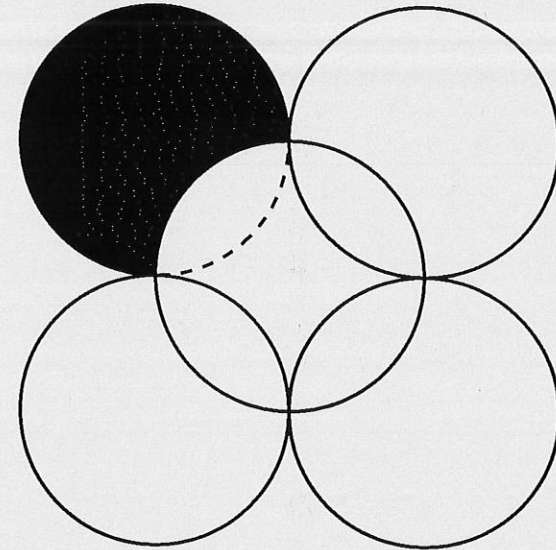


Question Nine :

[5]

E·C·L·I·P·S·E

Determine the area of the shaded shape which was constructed from five identical circles each of radius "R" arranged as shown to the right.



Question Ten : G·I·V·E M·E A S·I·N·E

[10] A, B and C are three points on the circumference of a circle, centre O.
 With the usual notation, $AB = c$, $AC = b$ and $BC = a$.
 The angles and sides are connected by the well known Sine Rule.

i.e.

The Sine Rule		
$\frac{a}{\sin A}$	$=$	$\frac{b}{\sin B}$
	$=$	$\frac{c}{\sin C}$

Prove that each of these ratios is equal to the diameter of the circle.

Question Eleven : **P·R·I·M·E R·E·S·U·L·T**

[15] Prove that

$$p^2 \bmod 2p = p$$

where p is any odd prime number.

NOTE : The "mod" function means "has a remainder upon division by".

EXAMPLES : "13 mod 5 = 3" means "13 has a remainder upon division by 5 of 3".

"13 mod 6 = 1" means "13 has a remainder upon division by 6 of 1".

"13 mod 7 = 6" means "13 has a remainder upon division by 7 of 6".

Question Twelve : **A·S E·A·S·Y A·S A B C**

[20] a , b , and c are positive real numbers with the property that;

$$ab + bc + ca = 1$$

Show that; $a + b + c \geq \sqrt{3}$

END OF PAPER

Martin Hansen, February 2003.

(with thanks to Martin Cropper for advice and suggestions)