

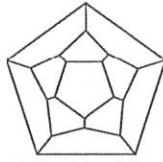
Arnold Hagger Prize Twenty Twelve

25th January

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This paper comprises two sections. Section A consists of 12 multiple choice questions, each correct answer scores 4 marks but 1 mark is taken off for any incorrect answer. If you leave the question unanswered then you score zero. Section B consists of 4 longer questions.

Calculators are allowed but not essential.



SECTION A

Please write your answers on the answer sheet provided

QUESTION 1

Evaluate

$$\sqrt{\frac{20^3 - 12^3}{20 - 12}}$$

- A 20 B 24 C 28 D 32 E 36

QUESTION 2

What month will it be in 2012 days time?

- R June S July T August U September V October

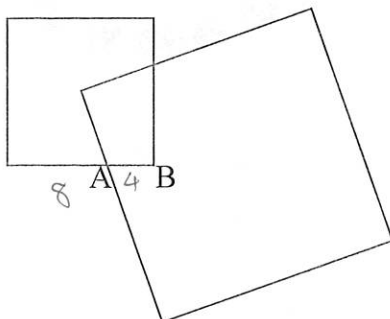
QUESTION 3

If, in a 20% off sale, a shopkeeper still makes a 12% profit on every item. What is his percentage profit on each item when sold at full price?

- N 32% O $33\frac{1}{3}\%$ P 35% Q 36% R 40%

QUESTION 4

The large square in the diagram has side length 20 and the smaller has side length 12. The corner of the large square is placed at the centre of the smaller square, what is the area of overlap if the length AB in the diagram is 4?



- O 24 P $12 + 8\sqrt{5}$ Q 36

- R $12 + 8\sqrt{10}$ S 48

QUESTION 5

Before 1971, Britain had a currency that had involved pounds, shillings and pence. There were 20 shillings (s) in a pound (£) and 12 pence (symbol d not p) in every shilling. How much change would you get from 12 pounds if you bought 2012 penny chews?

- L £2 13s 4d M £3 12s 4d N £3 14s 2d O £4 12s 3d P £4 13s 2d

QUESTION 6

It is ^A Bill and ^B Ben's birthday today. In 20 years time, ^A Ben will be 3 times the age ^B Bill was in 2000. In 12 years time, ^A Bill will be 3 times the age ^B Ben was in 1992. In what year was ^A Bill born?

- ^A D 1976 ^B E 1978 ^A F 1980 ^B G 1982 ^A H 1984

QUESTION 7

A group of 5 people contain knights and knaves. Knights always tell the truth and knaves always lie. They each give a statement,

- Exactly 1 of us is a knave
 Exactly 2 of us are knights
 Exactly 3 of us are knaves
 Exactly 4 of us are knights
 Exactly 5 of us are knaves

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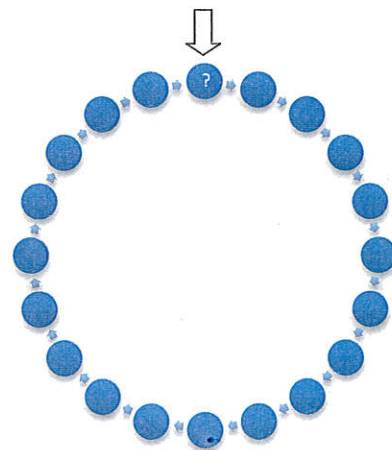
knave

How many of them are knights?

- H 1 I 2 J 3 K 4 L Need more information

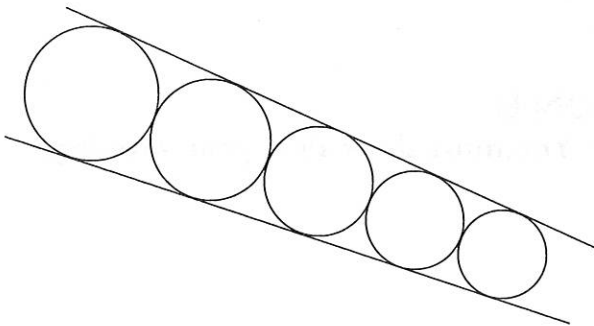
QUESTION 8

A restaurant produces 20 dishes and places them on a circular conveyor belt so that one can sample all of them without moving chairs. I like the first twelve dishes on the menu but do not care for the last eight. The dishes are arranged in the circle according the order on the menu. The 20th is between the 19th and 1st. I am seated at the table with one of the dishes in front of me. I take a bite and realise that it is one that I like. The belt rotates one place clockwise. What is the probability that I do not like the next dish?



- A 1/12 B 1/11 C 8/19 D 3/5 E 11/19

QUESTION 9



The five circles in the diagram have two common tangents as shown. The largest circle has radius 20cm and the smallest has radius 12cm. What is the area of the middle circle?

- G** 192π **H** 208π **I** 224π **J** 240π **K** 256π

QUESTION 10

How many letters are there in the English spelling of the number that is the absolute difference between the lowest positive integer with 20 letters in its English spelling and the lowest positive integer with 12 letters in its English spelling? For example, 101 is 'one hundred and one' so has 16 letters.

- G** 8 **H** 9 **I** 10 **J** 11 **K** 12

QUESTION 11

In this question; every letter represents a different digit; numbers with leading zeros are permissible. *NB: XYZ is the concatenation of the digits, e.g., 123, rather than their product, $1 \times 2 \times 3$.* If MDBJ is a perfect square, REP is prime and $MC \times RMW = A \times JMMB = 2012$. Which of the following sums is equal to JCA?

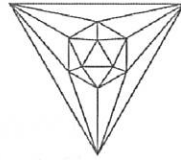
- E** $AD + DMJ - ADB$ **F** $AEM + CJM - RWA$ **G** $CER + RW - MWDB$
H $CJ + JRB - JAM$ **I** $MPJW + APP - PJM$

QUESTION 12

$$144 = 12 + 13 + 14 + \dots + 19 + 20$$

Which of the following numbers cannot be written as the sum of two or more consecutive positive integers?

- R** 2000 **S** 2012 **T** 2024 **U** 2036 **V** 2048



SECTION B

Please answer in the booklet provided. You must show all of your working.

QUESTION 13

We define $a * b$ by

$$a * b = \frac{a+b}{a-b}$$

Show that $20 * 12 = 4$ and evaluate

$$\left(20 * (12 * (20 * 12))\right) - \left(\left((20 * 12) * 20\right) * 12\right)$$

(6 marks)

QUESTION 14

My aunt has two children. Given one of them is boy, what is probability that she has two boys?

My aunt has two children. Given one of them is a boy born on a Wednesday, what is the probability that she has two boys?

(10 marks)

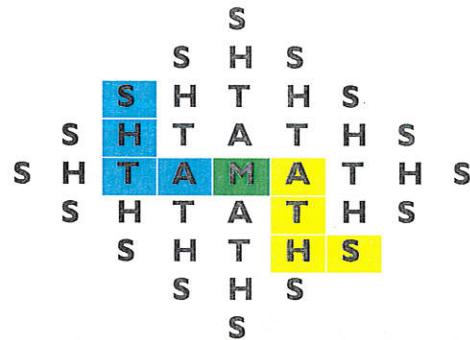
QUESTION 15

I have a number in the memory my calculator. If I take this number and add it to its reciprocal, I get a single digit positive integer. If I now square the original and add it to the reciprocal of this square, I again get a single digit positive integer larger than the previous result. I now cube the original number and add to it the reciprocal of this cube and press the 'equals' button. What number is now on the calculator's display?

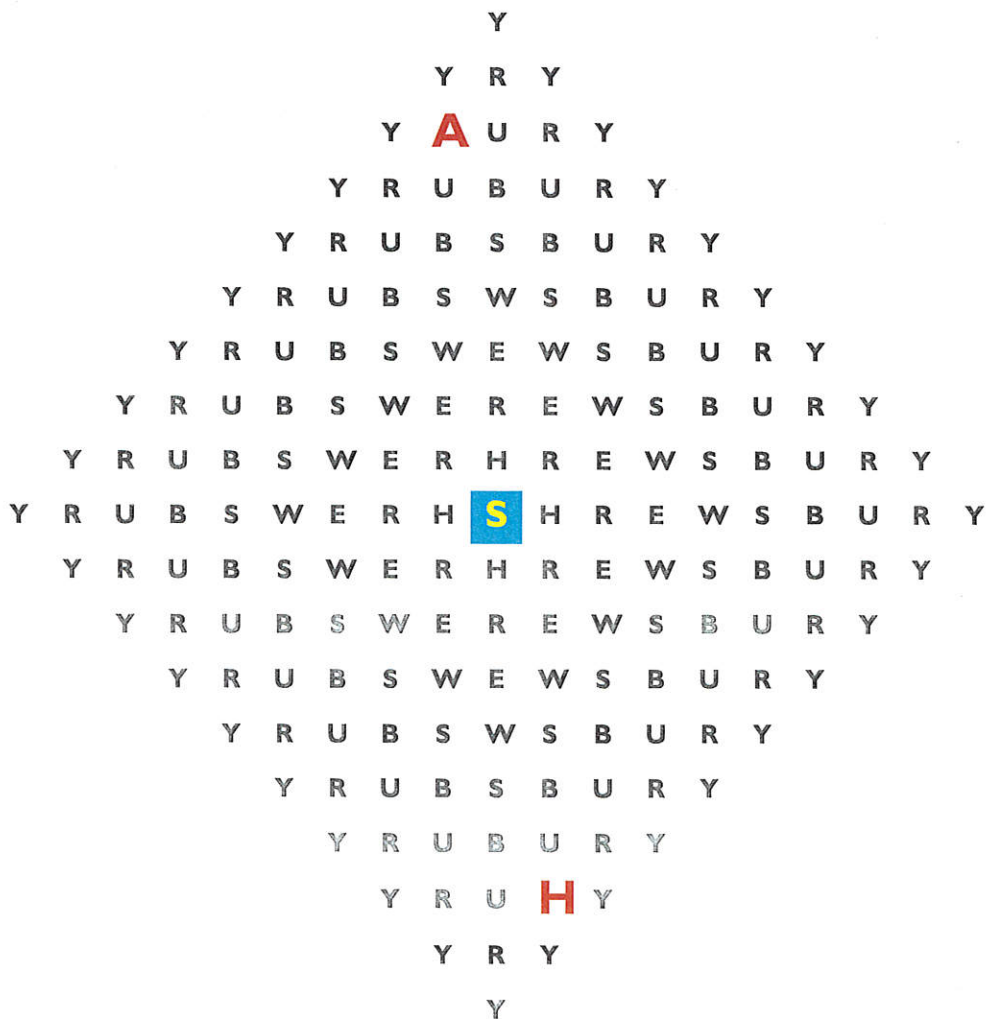
(10 marks)

QUESTION 16

The diagram shows two ways of spelling MATHS if one must start from the central M and collect the letters in order, moving to an adjacent letter at each step. Show that there are in fact 60 (incidentally, the lowest common multiple of 20 and 12) ways of doing so.



Starting from the central S, how many ways are there of spelling 'Shrewsbury' if you are only allowed to move to a neighbouring letter and collect the letters in the correct order. Notice the deliberate inclusion of an A(rnold) and a H(agger) to disrupt proceedings.



(14 marks)

END OF PAPER

Acknowledgements

With thanks to the UKMT and their mathematical challenges for inspiration for some of these questions and to JCA for his contributions in editing this paper.

CWO

25th January 2012