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Arnold Hagger Mathematics Competition

2014

This paper carries a total of 100 marks. There are:

4 Questions of 3 marks each

8 Questions of 5 marks each

6 Questions of 8 marks each

The marks for each question indicate the simplicity or intricacy of that question.

Answer as many questions as you can. Do not spend too long on any one question, if you become stuck.

Answers without explanation will receive few, if any, marks. Short, elegant solutions will gain more marks than long, contorted ones.

You may use a calculator in this paper.

Questions

1. What are the next two numbers in this sequence?
1 4 13 28 49

[3 Marks]

2. If you take the non-zero digits and place them as follows:

$$\frac{6729}{13458}$$

you will have a fraction equal to $\frac{1}{2}$.

Now rearrange the nine digits so as to form a fraction equivalent to $\frac{1}{4}$.

[3 Marks]

3. Let **E**, **H**, **I**, **M** and **S** be different single non-zero digits.
If **I** multiplied by **I** equals **ME**, and **ME** multiplied by **ME** equals **SHE**, how much is **SHE** worth?

[3 Marks]

4. A magic square is a square in which the digits from 1 to 9 are arranged, such that the sum of every row, column or diagonal equals 15. An example is shown below:

2	9	4
7	5	3
6	1	8

Now find an 'antimagic' square in which the digits are rearranged such that **none** of the sums of any row, column nor diagonal equals 15.

[3 Marks]

5. A boy is chosen at random and found to come from a family with two children.
a. What is the probability that the other child is a boy?

A certain family with two children is known to contain a boy.
b. What is the probability that it contains two boys?

[5 Marks]

6. A bath has two taps and a plug-hole. One of the taps on its own would fill the bath in ten minutes; the other would take a quarter of an hour. The plug-hole can empty a full bath in exactly seven and a half minutes.
If both taps are turned full on, and the plug-hole is left open, when (if ever) will an empty bath become full?

[5 Marks]

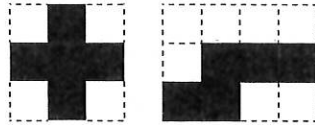
7. Find a triangle that can be cut into five triangles each similar to itself.
Similar triangles have the same shape (ie same angles) as each other, but may have different lengths (though in the same ratio).

[5 Marks]

8. A survey of 100 people were asked whether they had watched BBC, ITV or Sky programmes on a particular evening.
- Viewers who had watched the BBC during the evening numbered 44.
 - 43 said they had seen ITV programmes.
 - 28 said they had watched Sky programmes.
 - Another 23 people said they had not watched any television that evening.
 - 17 said they had seen BBC and ITV programmes.
 - 13 had seen BBC and Sky programmes.
 - Another 13 had seen ITV and Sky programmes.
- How many people watched programmes on all three television channels?

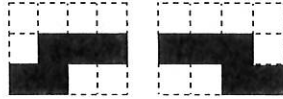
[5 Marks]

9. A pentomino is a shape that is made up of 5 identical squares that are joined together by common edges. Two examples are shown below:

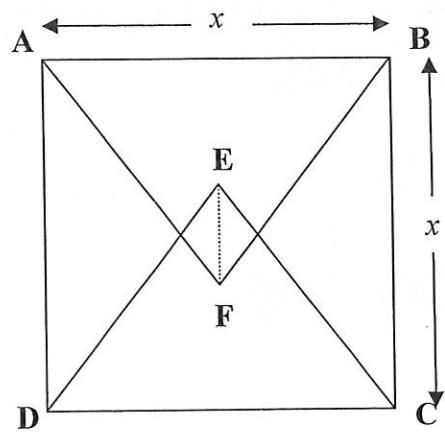


How many different pentominoes are there?

(Note: Any two pentominoes that are reflections or rotations of each other are only counted once, eg these two shown below are counted as being the same pentomino)



10. **ABCD** is a square of side x cm. **AFB** and **DEC** are equilateral triangles. Calculate the distance **EF** in terms of x .



[5 Marks]

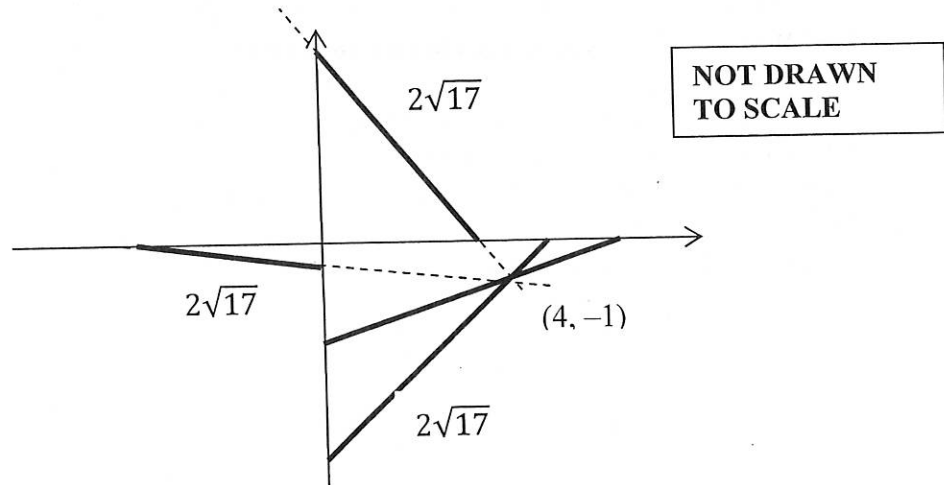
11. Each morning, my chauffeur drives me to the railway station where I catch the train to Shrewsbury. Each evening on my return home, at the same time as the train arrives at the station, my chauffeur arrives to take me home.

Yesterday, I caught an earlier train, arriving back at the station an hour earlier than usual. Not wishing to cause a fuss, I began to walk home. On the way I met my chauffeur on the road, as he was on the way to meet my normal train, and he drove me the rest of the way home. This meant that I arrived home twenty minutes earlier than usual.

How long did I spend walking?

[5 Marks]

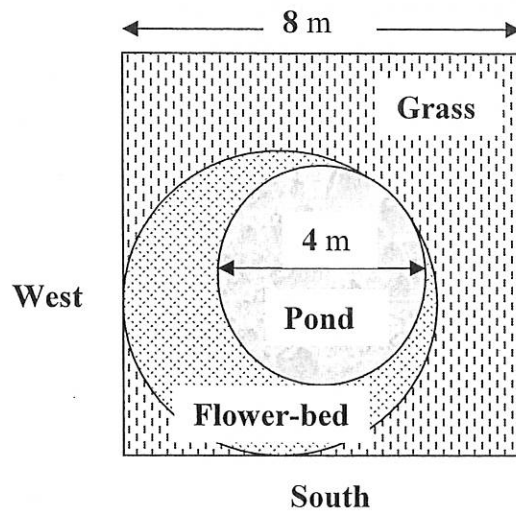
12. There are four straight lines through the point $(4, -1)$, each of whose line segment intercepted by the two axes has a length equal to $2\sqrt{17}$, as shown below:



Find the equation of any one of these lines.

[5 Marks]

13. A small square garden has side length of 8 m, and has a small pond of diameter 4 m in the centre. The flower-bed around the pond is a circle that just touches the western and southern edges of the garden perimeter, and also just touches the central pond, as shown in the diagram.



If the remainder of the garden that is not the pond or flower-bed is of grass, what is the ratio of flowerbed to grass?

[8 Marks]

14. I have a solid wooden cube with **whole number dimensions**. I paint the entire surface of the cube red. Then, with slices parallel to the faces of the cube, I cut the cube into $1 \times 1 \times 1$ cubes. Let:
- x = the number of the small cubes completely free of paint,
 - y = the number of the small cubes painted red on only one side, and
 - z = the number of the small cubes painted red on two sides.

If y plus z is exactly 33% of x , what was my original cube size?

[8 Marks]

15. I have three containers which hold, when filled to the brim, exactly 8, 5 and 3 litres respectively. By starting with the largest one full, denoted as (8, 0, 0), I can pour from one to the other, and without wasting a drop, eventually divide the water into two equal portions, denoted as (4, 4, 0).

To do this in eight operations, the sequence could run as follows:

	Start	(8, 0, 0)	
1	Pour into C from A	(5, 0, 3)	
2	Pour into B from C	(5, 3, 0)	
3	Pour into C from A	(2, 3, 3)	
4	Pour into B from C	(2, 5, 1)	
5	Pour into A from B	(7, 0, 1)	
6	Pour into B from C	(7, 1, 0)	
7	Pour into C from A	(4, 1, 3)	
8	Pour into B from C	(4, 4, 0)	Finish

It is possible to achieve the same result in less than eight operations. What is the least number of operations possible, and what is the sequence of operations?

[8 Marks]

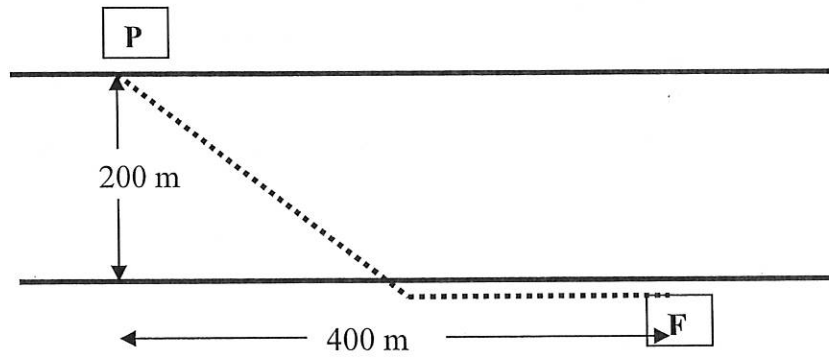
16. A man walks from the village of Startington to Endington. On the first day he covers $\frac{1}{10}$ of the journey. On the second day he goes $\frac{2}{3}$ of the distance covered on the first day. He continues in this fashion, alternating days on which he covers $\frac{1}{10}$ of the distance still to be travelled, with days on which he covers $\frac{2}{3}$ of the **total** distance already travelled. After seven days of walking the route, he calculates that only another **15** miles need be walked before he reaches Endington.

How far is it from Startington to Endington?

17. Zöe decided to sell her collection of books.
To Alfred, she sold 2 books and one fifth of what was left.
Later to Beatrice she sold 6 books and one fifth of what then remained.
If she sold more books to Alfred than to Beatrice, what was the least possible number of books in her original collection?

[8 Marks]

18. A power station, **P**, is on one bank of a straight river 200 m wide, and supplies electricity to a factory, **F**, on the opposite bank 400m downstream from **P**. The power cable has to be taken across the river. To lay the cable underwater costs £5 per metre, but on land it costs £3 per metre.



What path should be chosen so that the cost is minimized?

[8 Marks]

END OF QUESTIONS