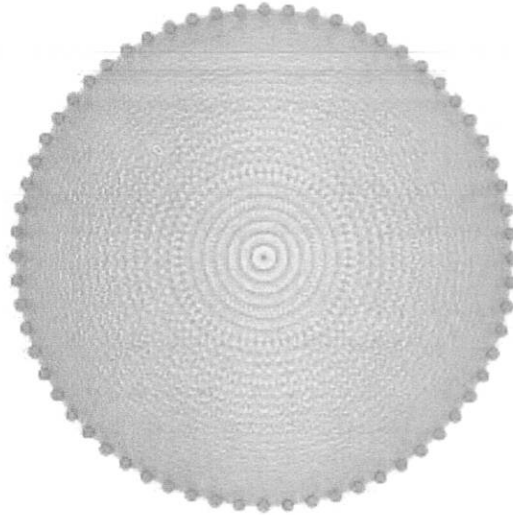


Arnold Hagger Prize 2016

20th January 2016



[A complete graph with 64 vertices and 2016 edges]

I am very grateful to Mr Armstrong for allowing me to write this paper in such an auspicious year. $2016 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$. This factorisation may prove useful. I set the paper back in $2012 = 2 \times 2 \times 503$; people still recall that the answer to last question was in fact 2012. We shall see what this year's paper brings – maybe there will be a twist in the tale.

Good luck! I hope that there are some questions that you can attempt even if you do not get the full solution. Calculators are **not** allowed. You may attempt as many questions as you can in 90 minutes; please write your answers in the spaces provided.

Good luck!

CWO

QUESTION ONE

Arrange the digits 2, 0, 1 and 6 and an equals sign to make a mathematically correct expression. Some examples are $6^0 = 2 - 1$ or $6^0 + 1 = 2$, $26^0 = 1$, $62^0 = 1 \dots$ Any mathematical operation may be used. Two marks for a mathematically correct expression. Three marks if the digits 2, 0, 1 and 6 are in that order in your expression.

[3 marks]

QUESTION TWO

2016 appears in the sequence 1, 3, 6, 10, 15, 21, 1953 was the last year to do this. What is the next year to appear in this sequence?

[3 marks]

QUESTION THREE

Write

$$\sqrt{20\% \times 1\% \times 20\% \times 16\%}$$

as a single percentage.

[3 marks]

QUESTION FOUR

What is the last digit of $16^{20} - 20^{16}$?

[3 marks]

QUESTION FIVE

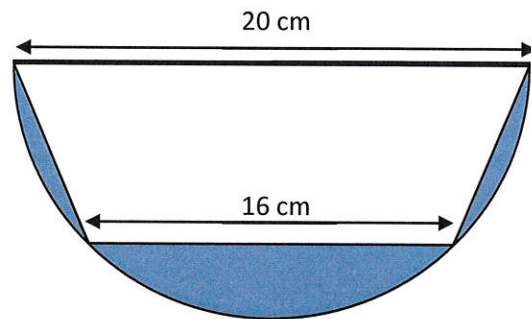
Edward VII became King of England on 20th January 1936. Today is Wednesday in case you have forgotten. On which day of the week did Edward VII become king?

[5 marks]

QUESTION SIX

Four points on the circumference of a semicircle, with diameter 20cm, can be joined to form an isosceles trapezium with parallel sides of length 20cm and 16cm as shown in the diagram

Find the exact value of the shaded area in cm^2



[5 marks]

QUESTION SEVEN

Zebedee enters a running race. He keeps up with the leaders for the first few kilometres, averaging 20kph over this distance. He then starts to fade and ran the remaining 12km at an average speed of 16kph. He realised at the end of the race that his average speed for the entire race was 17kph. How long was the race?

[5 marks]

QUESTION EIGHT

Aloysius and his five brothers are aged 3, 5, 7, 9, 11 and 13. One afternoon, two of his brothers whose ages sum to 16 went to the park; another two brothers, both younger than 10, went to play computer games with the girls next door. That left Aloysius looking after his sick 5 year old brother. How old is Aloysius?

[5 marks]

QUESTION NINE

Haggerland has amongst its currency, blue coins, red coins and silver coins. Greta has 20 blue coins and 16 red coins in her purse. She wishes to exchange these for silver coins. She is miles from any bank or person who can help. However she finds two machines that offer the exactly the same rate of exchange with no commission. The first machine can change exactly two red coins for a silver coin and a blue coin. The second machine will exchange three blue coins for one silver coin and a red coin. Assuming that the machines have a sufficient number of coins, for how many silver coins can she exchange the contents of her purse?

How many silver coins might you expect to get with 2016 blue and 2016 red coins at your disposal (and two very well stocked machines)? You do not have to show the exchanges required to achieve this.

[7 marks]

QUESTION TEN

I am meeting a friend in town at 8.30pm. My very accurate analogue wrist watch is showing the time to be 8.16pm (2016 on a 24hr clock) with the second hand pointing to 12. I make good time and am slightly early for my meeting. I look at my wrist watch when I reach my destination and notice that the hour and minute hands are at exactly 90 degrees. How long did my journey take me? Give your answer in an exact form.

[7 marks]

QUESTION ELEVEN

In this question; every letter represents a different digit; numbers with leading zeros are permissible. XYZ is the concatenation of the digits, e.g., 123 , rather than their product, $1 \times 2 \times 3$.

$JMMB \times MC = 2016$. MHH and JVL are both factors of 2016. $MDBJ$ is the product of three consecutive integers and the sum of the digits of $SLMS$ is prime.

What is the value of $J + C + A$?

[7 marks]

QUESTION TWELVE

How many ways can 20 identical objects be shared amongst 16 people so that every person receives an odd number of objects?

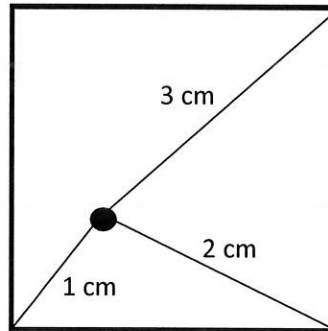
[7 marks]

QUESTION THIRTEEN

A point is chosen in the interior of a given square such that it is 1 cm away from the first corner, 2 cm away from a second corner and 3 cm away from a third.

What is the distance from this point to the fourth corner?

What is the area of the square?



[10 marks]

QUESTION FOURTEEN

To pass the time on an exceedingly dull coach ride, Xerxes and Yvonne play a game. Xerxes picks a whole number between 1 and 50 and Yvonne must try to work it out through asking a series of questions to which Xerxes may only answer “yes” or “no”. Yvonne starts with “Is it greater than 25?”... that obviously doesn’t narrow it down too much... “Is it an even number?” she asks... OK, “Is it a multiple of 3?”... “Is the number exactly divisible by 5?”... Yvonne took a deep breath to try to process all the responses. She smiled – “Is it a square number?”. “No” replied Xerxes. Yvonne then correctly announces Xerxes number and explains her faultless logic.

What number did Xerxes pick?

[10 marks]

QUESTION FIFTEEN

A number n is said to be 'pleasant' if there exists an integer m with exactly four positive divisors (including 1 and m) and these four divisors add up to n . For example 40 is pleasant since 27 has four divisors, 1, 3, 9 and 27 and these add up to 40. Explain why 2016 is a pleasant number.

[10 marks]

QUESTION SIXTEEN

If $x > 0$ and $x + \frac{1}{x} = 3$,

(a) show that:

$$x^2 + \frac{1}{x^2} = 7.$$

(b) Find the value of:

$$x^8 - 4x^4 - \frac{4}{x^4} + \frac{1}{x^8}$$

[10 marks]