2021

THE ARNOLD HAGGER MATHEMATICS PRIZE COMPETITION

THURSDAY 6th May

7.15pm - 8.45pm

At Shrewsbury School

Calculators MAY be used

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460955058223172535940812848111745028

Arnold Hagger Mathematics Prize Competition 2021

* The fourteen questions may be answered in any order.
* Make your methods of solution clear by including all working and reasoning.
* The marks allocated to each question is shown - either **[5]** or **[10]** marks.
* **Calculators MAY be used**.

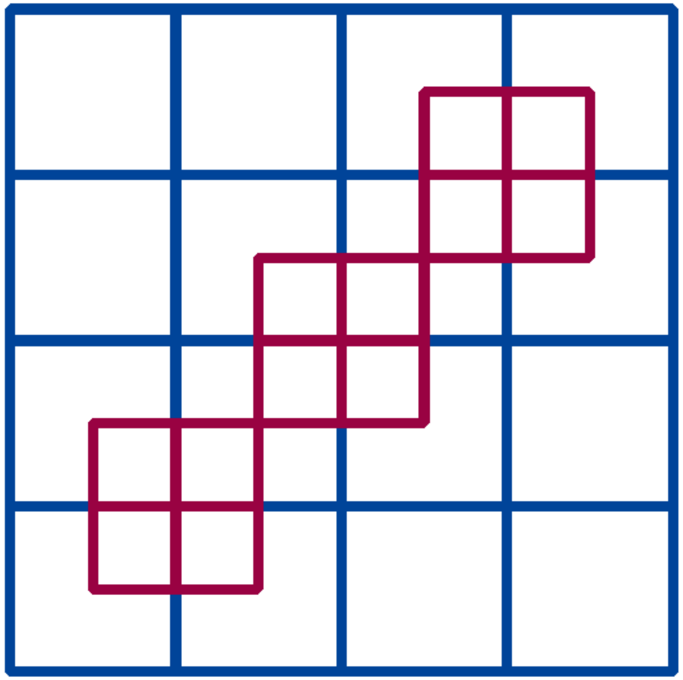
Question One : **B·O·X·I·N·G C·L·E·V·E·R**

[5] A cuboid has faces of areas 12 cm², 8 cm² and 6 cm².
What is its volume ?

Question Two : **E·A·S·Y A·S A·B·C**

[5] Solve for A, B, C.
The digits are distinct
and positive.

$$\begin{array}{r} AA \\ BB \\ +\ CC \\ \hline ABC \\ \hline \end{array}$$

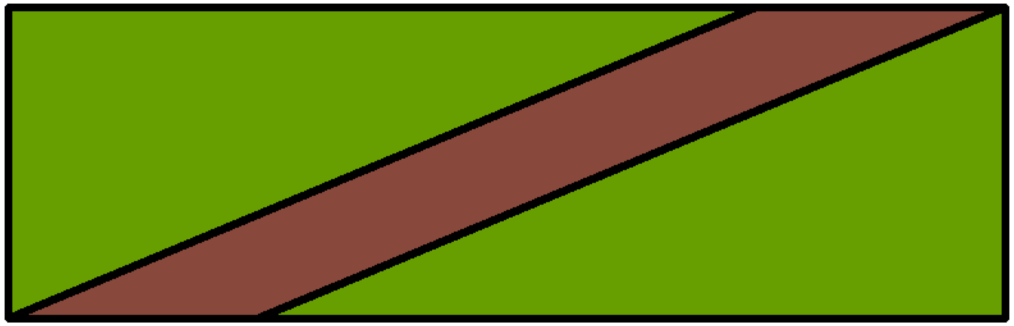


Question Three : **S·Q·U·A·R·E E·Y·E·D**

[5] How many squares are there in the diagram ?

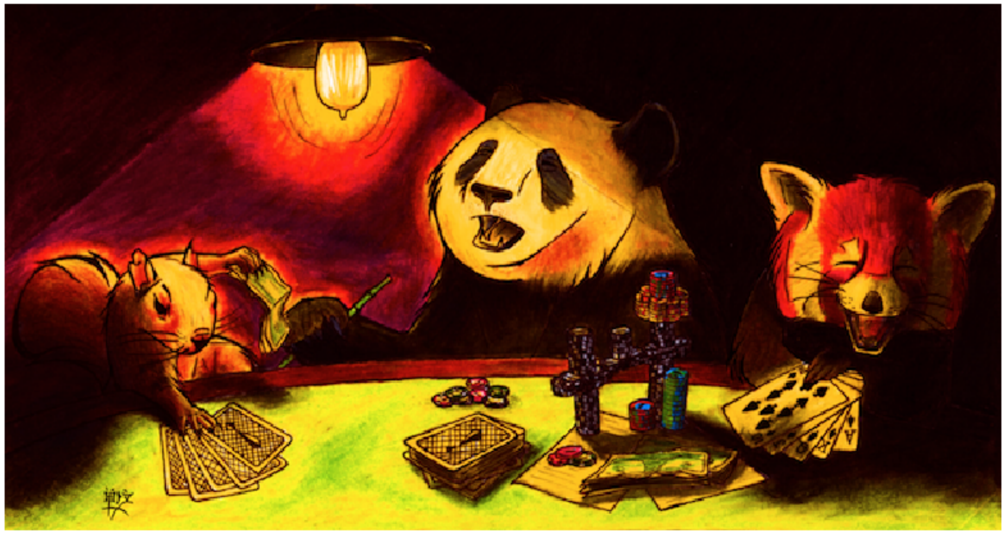
Question Four : **R·O·A·D T·O S·U·C·C·E·S·S**

[5] A straight road is laid across a rectangular plot of land, 160 m by 50 m, in the manner shown in the diagram.
Its area is found to be exactly one quarter of that of the original plot.
What is its width ?



Question Five : **W·I·N·N·E·R·S A·N·D L·O·S·E·R·S**

[5] A Squirrel, S, a Panda, P, and a Fox, F, are playing poker.
The sums of money in the hands of the three card players S, P and F are in the ratio 4:3:2 respectively at the beginning and in the ratio 3:2:1 respectively at the end of the game.
What is the percentage change in the money held by each player at the game's conclusion ?



Question Six : W·H·O·P·P·E·R

[5] Let me tell you about Poisson, my pet fish. Her head is 9 cm long, her body is as long as her head and her tail, and her tail is as long as her head and half her body.

How long is Poisson ?

Question Seven : J·U·G·G·L·I·N·G

[10] I have three jugs *A*, *B* and *C*, which hold, when filled to the brim, exactly 8, 5 and 3 litres respectively. The number of litres in each jug is denoted (*A*, *B*, *C*). By starting with the largest one full, denoted as (8, 0, 0), I can pour from one to the other and, without wasting a drop, eventually divide the water into two equal portions, denoted as (4, 4, 0).

To do this in eight operations, the sequence could run as follows:

	Start	(8, 0, 0)
1	Pour from *A* into *C*	(5, 0, 3)
2	Pour from C into *B*	(5, 3, 0)
3	Pour from *A* into *C*	(2, 3, 3)
4	Pour from *C* into *B*	(2, 5, 1)
5	Pour from *B* into *A*	(7, 0, 1)
6	Pour from *C* into *B*	(7, 1, 0)
7	Pour from *A* into *C*	(4, 1, 3)
8	Pour from *C* into *B*	(4, 4, 0)
	Finish	

It is possible to achieve the same result in fewer than eight operations.
What is the least number of operations, and what is the sequence ?

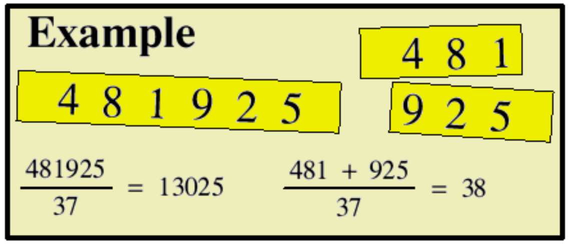
Question Eight : F·O·O·D F·O·R T·H·O·U·G·H·T

[5] After a banquet at an Indian restaurant, there were 65 serving dishes to wash up.
"How many guests were at the banquet?" asked the manager.
"I didn't have time to count them" the waiter replied, "but I do know that there was one dish of rice for every two people, there was a vegetable dish for every three people, and a large curry dish for every four people".

How many guests were there at the banquet ?

Question Nine : **M·A·G·I·C T·H·I·R·T·Y S·E·V·E·N**

[10] A six digit number, written on card, is divisible by 37. The card and the number on it are cut in half to make two three digit numbers.



Prove that the sum of these two three digit numbers is also divisible by 37.

Question Ten : **P·E·R·F·E·C·T S·Q·U·A·R·E**

[10] Find all integers n for which $n^2 + 20n + 11$ is a perfect square.

Question Eleven : **G·O·O·D·B·Y·E P·Y·T·H·A·G·O·R·A·S**

[10] Given any three prime numbers, p, q and r, which need not be distinct, prove that

$$p^2 + q^2 \neq r^2$$

Question Twelve : **I·N·C·L·I·N·E·D T·O B·E S·Q·U·A·R·E**

[10] The diagram, which is not drawn to scale, shows square $OABC$ which has one vertex at the origin.
Vertex A has the coordinates $(1, \sqrt{2})$
Find the exact coordinates of vertex B and vertex C.

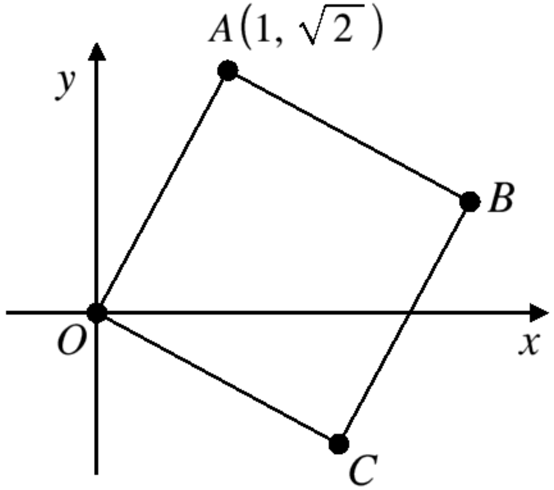
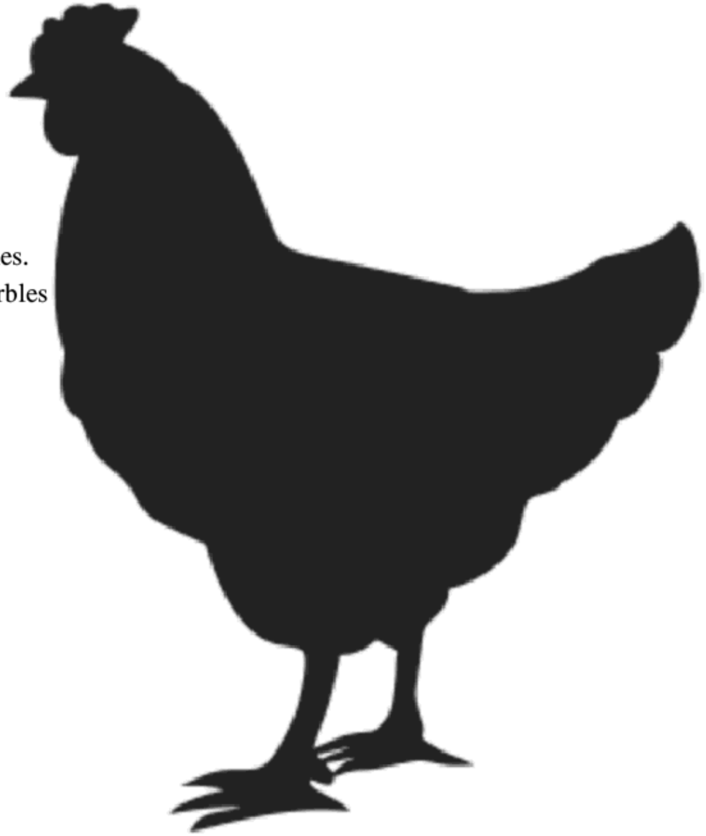


Diagram NOT accurately drawn

Question Thirteen : **A·S R·A·R·E A·S H·E·N·S' T·E·E·T·H ?**

[5] How many three digit numbers are divisible by three and have the additional property that the sum of their digits is four times the middle digit ?



Question Fourteen : **M·I·R·A·C·L·E M·A·R·B·L·E·S**

[10] The Miracle Marble Manufacturing Company makes orange and purple marbles.
A bag of their marbles may contain any combination of orange and purple marbles (including all orange and all purple) and all combinations are equally likely, given any fixed total number of marbles.
Henry bought a bag of their marbles and pulled one out at random.
It was purple.
What is the probability that, if he pulled a second marble at random without replacing the first, it would also be purple ?

END OF PAPER

Martin Hansen, May 2021

With thanks to Ian Payne for his enthusiastic support for the competition
to Jerome Armstrong and Dr Charlie Oakley for proofreading the questions
and to Sara Luzny for producing a magnificent poster for the event