## 11.1 Later Date Revision

When two geometric objects are described as being *similar* it means that they have the same *shape* but not necessarily the same *size*; one is an enlargement of the other. All the lengths of one are more than (or less than) the corresponding lengths of the other by the same multiplicative length scale factor, *lsf*. Also, any area of one is more than (or less than) the corresponding area of the other by the same multiplicative area scale factor, *asf*. The same is true of any corresponding volumes. In summary, between two shapes described as being similar, there can be up to three different scale factors in play;

- Length scale factor, *lsf*
- Area scale factor, *asf*
- Volume scale factor, *vsf*

Of great use is the mathematical relationship between these three scale factors. Given any one, the other two can be found as follows;

- Finding *lsf* from either the *asf* or the *vsf*

$$\text{length scale factor} = \sqrt{\text{area scale factor}}$$

$$\text{length scale factor} = \sqrt[3]{\text{volume scale factor}}$$

- Finding *asf* from either *lsf* or *vsf*

$$\text{area scale factor} = (\text{length scale factor})^2$$

$$\text{area scale factor} = \left(\sqrt[3]{\text{volume scale factor}}\right)^2$$

- Finding *vsf* from either *lsf* or *asf*

$$\text{volume scale factor} = (\text{length scale factor})^3$$

$$\text{volume scale factor} = \left(\sqrt{\text{area scale factor}}\right)^3$$

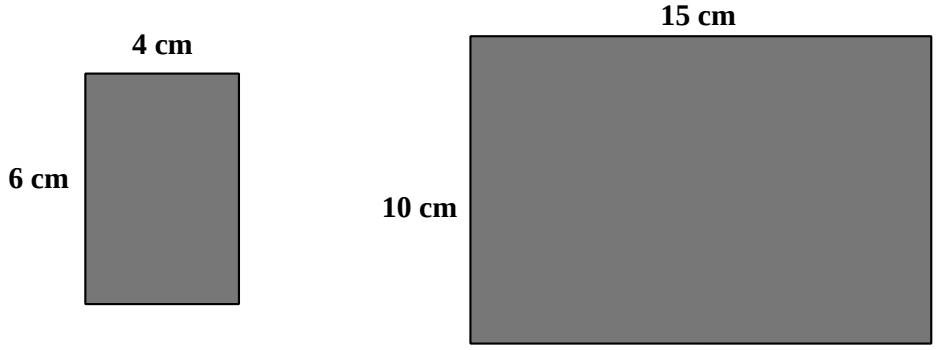
Spotting similar shapes in geometric problems often provides a particularly easy method of solution side stepping more complicated answers that use trigonometry and angle properties such as alternate or corresponding angles. Even just identifying that two triangles are similar is often the key step, after which using nothing more than the length scale factor between them is enough to crack a seemingly difficult question.

Easier questions present the similar shapes in the same orientation but, as the following example demonstrates, this is not always the case.

**11.2 Orientation Examples**

**( i )** The two rectangles shown below are similar even although

$$\frac{10}{6} \neq \frac{15}{4}$$




Complete the following to explain the apparent anomaly.

——— = ——— Which both cancel down to ———

∴ The two rectangles are indeed similar.

**[ 2 marks ]**

**( ii )** Is a Δ with sides 5, 3 and 4 cm similar to one with sides 12, 9 and 15 cm ? Give a reason for your answer.

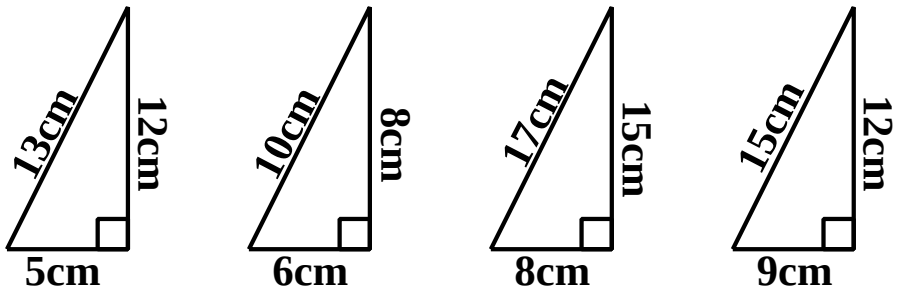
Yes, because $\frac{15}{5}$ = ——— = ———

which all cancel down to ———

∴ The two triangles are similar.

**[ 2 marks ]**

**( iii )** Which two of the following triangles are similar ?




Explain your answer.

——— = ——— = ——— which all cancel down to ———

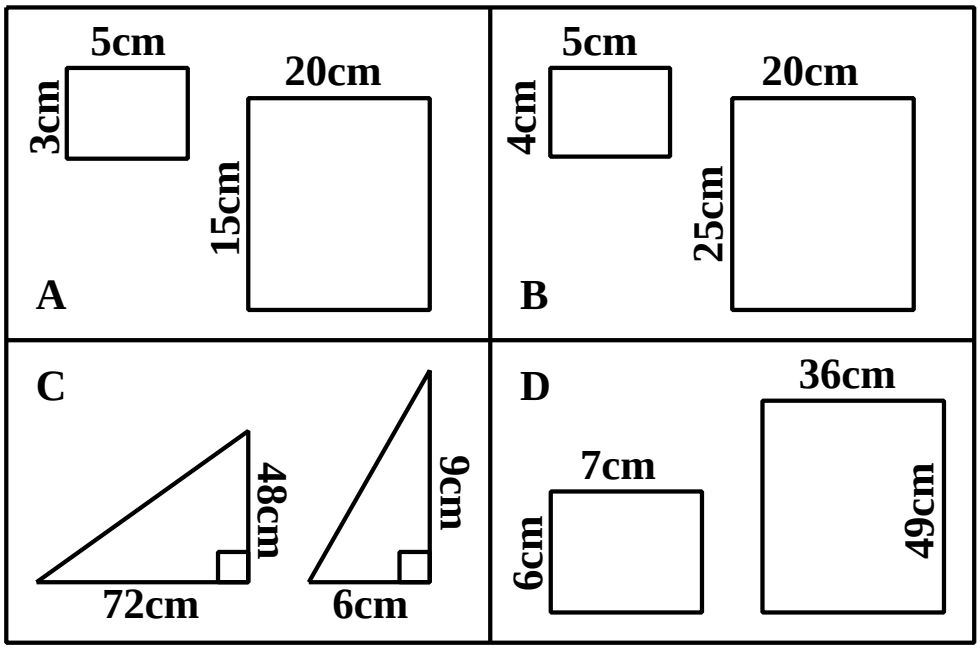
∴

**[ 2 marks ]**

**11.3 Exercise**

Marks Available : 50

**Question 1**




For each pair of shapes, complete the following to determine in each case if the pair is similar or not.

**A**

Not similar because ——— ≠ ———

and also ——— ≠ ———

**[ 2 marks ]**

**B**

Similar because ——— = ——— which both cancel to ———

**[ 2 marks ]**

**C**

Similar because ——— = ——— which both cancel to ———

**[ 2 marks ]**

**D**

because ——— ———

**[ 2 marks ]**

**Question 2**

The ***area*** scale factor between two similar solids is $\frac{49}{25}$

**( i )** What is the ***length*** scale factor between the two solids ?

**[ 1 mark ]**

**( ii )** What is the ***volume*** scale factor between the solids ?

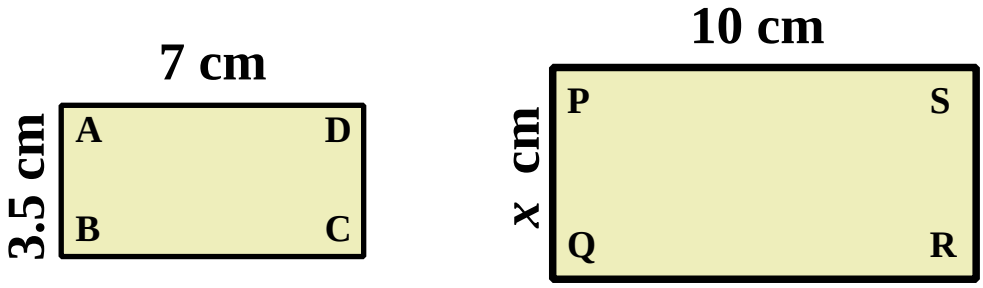
**[ 1 mark ]**

**( iii )** If the smaller has a volume of 250cm$^3$ what is the volume of the larger ?

**[ 1 mark ]**

**Question 3**

Rectangle *ABCD* is similar to rectangle *PQRS*.




Find the length of side *x* assuming that the 7 cm and 10 cm sides pair off.

**[ 2 marks ]**

**Question 4**

One cube has a volume which is 64 times greater than the volume of another cube.

**( i )** Explain why all cubes are similar.

**[ 1 mark ]**

**( ii )** What is the ***length scale factor***, greater than 1, between the cubes ?

**[ 1 mark ]**

**( iii )** What is the ***area scale factor***, greater than 1, between the cubes ?

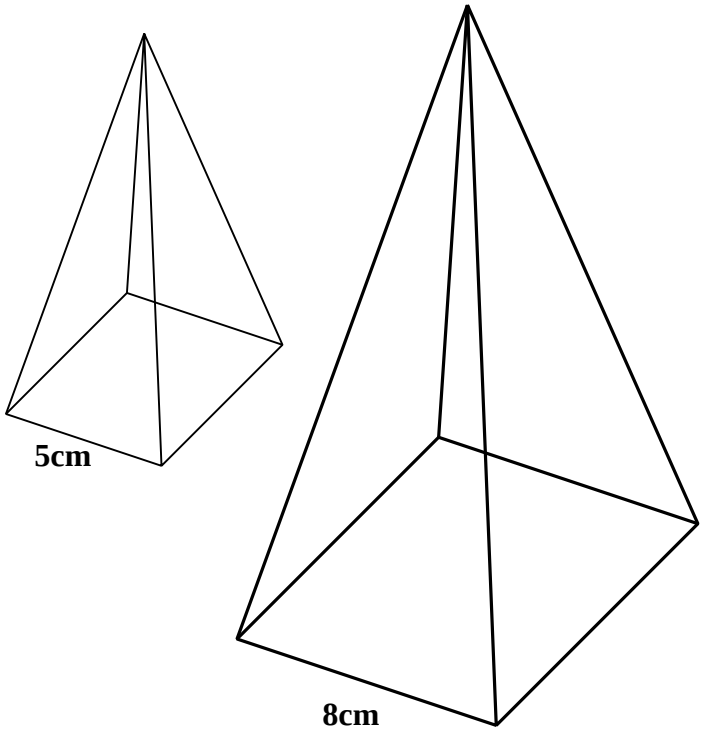
**[ 1 mark ]**

**( iii )** If the smaller cube has a total surface area of 20cm$^2$ what is the total surface area of the larger cube ?

**[ 1 mark ]**

**Question 5**

A couple of similar square based pyramids are shown below.




**( i )** What is the ***length scale factor***, greater than 1, of the similarity ?

**[ 1 mark ]**

**( ii )** What is the ***length scale factor***, less than 1, of the similarity ?

**[ 1 mark ]**

**( iii )** What is the ***area scale factor***, greater than 1, of the similarity ?

**[ 1 mark ]**

**( iv )** What is the ***area scale factor***, less than 1, of the similarity ?

**[ 1 mark ]**

**( v )** What is the ***volume scale factor***, greater than 1, of the similarity ?

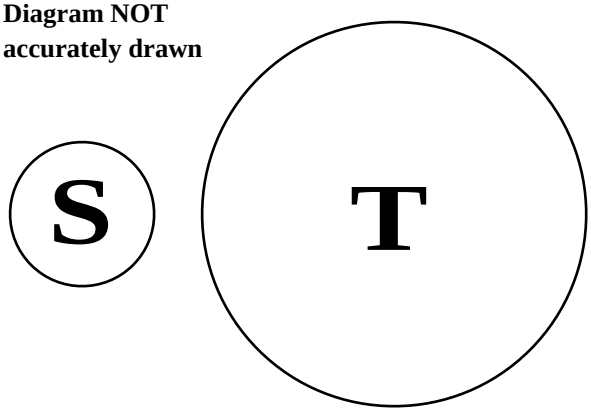
**[ 1 mark ]**

**( vi )** What is the ***volume scale factor***, less than 1, of the similarity ?

**[ 1 mark ]**

**Question 6**

**Diagram NOT accurately drawn**




The area of circle **S** is 4 $cm^2$.
The radius of circle **T** is three times the radius of circle **S**.

Work out the area of circle **T**.

**[ 2 marks ]**

**Question 7**

True or false ?

**( a )** All equilateral triangles are similar.

**[ 1 mark ]**

**( b )** All cubes are similar.

**[ 1 mark ]**

**( c )** All triangles with sides 3cm, 4cm and 5cm are similar.

**[ 1 mark ]**

**( d )** All rectangles are similar.

**[ 1 mark ]**

**( e )** An A5 piece of paper paper is similar to an A4 piece of paper.

**[ 1 mark ]**

**( f )** A football is similar to a rugby ball.

**[ 1 mark ]**

**( g )** All square based pyramids are similar.

**[ 1 mark ]**

**( h )** All dodecahedrons are similar.

**[ 1 mark ]**

**( i )** All circles are similar.

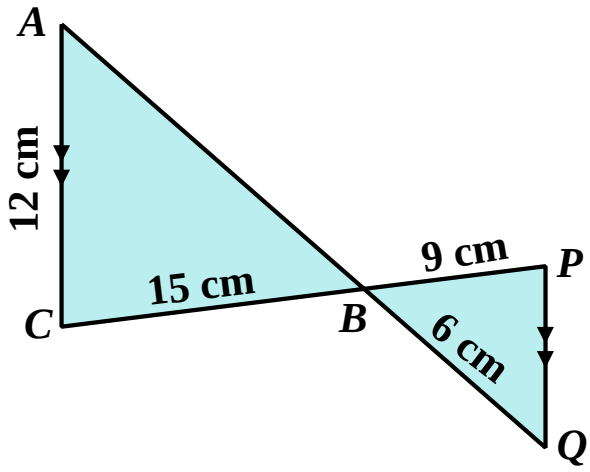
**[ 1 mark ]**

**( j )** All parabolas are similar.

**[ 1 mark ]**

**Question 8**

The two triangles in the diagram are similar with *AC* parallel to *PQ*.




Think about alternate angles. ( *Z* - angles )

**( i )** Is $\angle PQB$ the same size as $\angle BCA$ or $\angle BAC$ ?

**[ 1 mark ]**

**( ii )** Is the length scale factor between the similar triangles $\frac{15}{9}$ or $\frac{15}{6}$ ?

**[ 1 mark ]**

**( iii )** Find the length of *AB* and also the length of *PQ*

**[ 2 marks ]**

**Question 9**

*GCSE Examination Question from June 2021, Paper 2H, Q20 (Edexcel)*

Mathematically similar wooden blocks are made in a workshop.

There are small blocks and there are large blocks.

The volume of each small block is 300 $cm^3$

Given that

the surface area of each small block : the surface area of each large block

$$= 25 : 36$$

work out the volume of each large block.

**[ 3 marks ]**

**Question 10**

*GCSE Examination Question from January 2022, Paper 1H, Q17 (Edexcel)*

**A** and **B** are two similar vases.

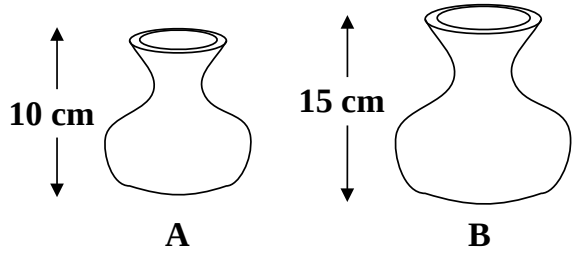



**Diagram NOT accurately drawn**

Vase **A** has height 10 cm

Vase **B** has height 15 cm

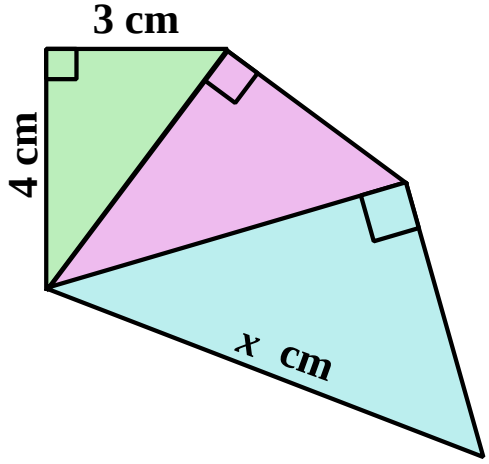
The difference between the volume of vase A and the volume of vase B is 1197 $cm^3$

Calculate the volume of vase **A**

**[ 4 marks ]**

**Question 11**

The three triangles in the figure below are all similar.




Find the length marked *x*.

**[ 4 marks ]**