

Lesson 2

GCSE Mathematics Ratio and Similarity

2.1 Starter : Squaring Vulgar Fractions

Here are three examples to illustrate how to square vulgar fractions.

Study the three examples:

Example #1

With all the steps shown;

$$\begin{aligned}\left(\frac{7}{10}\right)^2 &= \left(\frac{7}{10}\right) \times \left(\frac{7}{10}\right) \\ &= \frac{7 \times 7}{10 \times 10} \\ &= \frac{7^2}{10^2} \\ &= \frac{49}{100}\end{aligned}$$

Example #2

Trying to do it faster by missing out some steps;

$$\begin{aligned}\left(\frac{3}{4}\right)^2 &= \frac{3^2}{4^2} \\ &= \frac{9}{16}\end{aligned}$$

Example #3

At speed : Doing it all mentally, just writing down the answer;

$$\left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

For You To Do

Try to do these nine questions super quick without writing down any working;

$$\text{(i)} \quad \left(\frac{4}{5}\right)^2 = \frac{\quad}{25} \quad \text{(ii)} \quad \left(\frac{7}{3}\right)^2 = \frac{49}{\quad} \quad \text{(iii)} \quad \left(\frac{6}{11}\right)^2 = \frac{\quad}{\quad}$$

$$\text{(iv)} \quad \left(\frac{1}{8}\right)^2 = \frac{\quad}{\quad} \quad \text{(v)} \quad \left(\frac{10}{9}\right)^2 = \frac{\quad}{\quad} \quad \text{(vi)} \quad \left(\frac{5}{7}\right)^2 = \frac{\quad}{\quad}$$

$$\text{(vii)} \quad \left(\frac{11}{6}\right)^2 = \frac{\quad}{\quad} \quad \text{(viii)} \quad \left(\frac{3}{1}\right)^2 = \frac{\quad}{\quad} \quad \text{(ix)} \quad \left(\frac{13}{14}\right)^2 = \frac{\quad}{\quad}$$

[9 marks]

2.2 Starter : Square-Rooting Vulgar Fractions

Here are two examples of how to square root vulgar fractions.

Study the two examples:

Example #1

With one step of working shown;

$$\begin{aligned}\sqrt{\left(\frac{81}{100}\right)} &= \frac{\sqrt{81}}{\sqrt{100}} \\ &= \frac{9}{10}\end{aligned}$$

Example #2

At speed : Doing it all mentally, just writing down the answer;

$$\sqrt{\left(\frac{9}{121}\right)} = \frac{3}{11}$$

For You To Do

Try to do these eight questions super quick without writing down any working;

$$(i) \sqrt{\left(\frac{16}{25}\right)} = \frac{\quad}{5} \qquad (ii) \sqrt{\left(\frac{49}{100}\right)} = \frac{7}{\quad}$$

$$(iii) \sqrt{\left(\frac{25}{36}\right)} = \frac{\quad}{\quad} \qquad (iv) \sqrt{\left(\frac{1}{9}\right)} = \frac{\quad}{\quad}$$

$$(v) \sqrt{\left(\frac{4}{1}\right)} = \frac{\quad}{\quad} \qquad (vi) \sqrt{\left(\frac{81}{16}\right)} = \frac{\quad}{\quad}$$

$$(vii) \sqrt{\left(\frac{400}{49}\right)} = \frac{\quad}{\quad} \qquad (viii) \sqrt{\left(\frac{196}{225}\right)} = \frac{\quad}{\quad}$$

[8 marks]

If you were good enough to get this far...

Challenge:

$$\left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{\quad}{\quad}$$

[2 marks]

2.3 The Connection between lsf and asf

Key Fact #1

To find the lsf if you know the asf

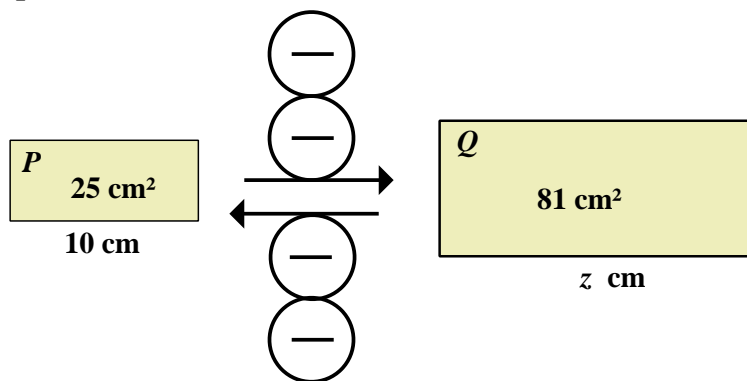
$$\text{length scale factor} = \sqrt{\text{area scale factor}}$$

Key Fact #2

To find the asf if you know the lsf

$$\text{area scale factor} = (\text{length scale factor})^2$$

2.4 Example

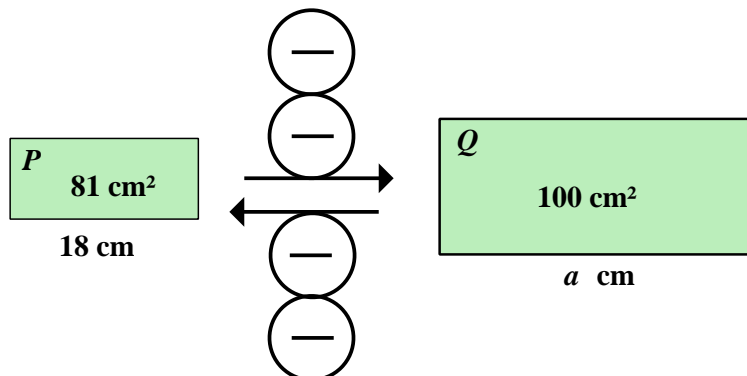


Rectangles P and Q are similar.
Find the length marked z

[3 marks]

2.5 Exercise

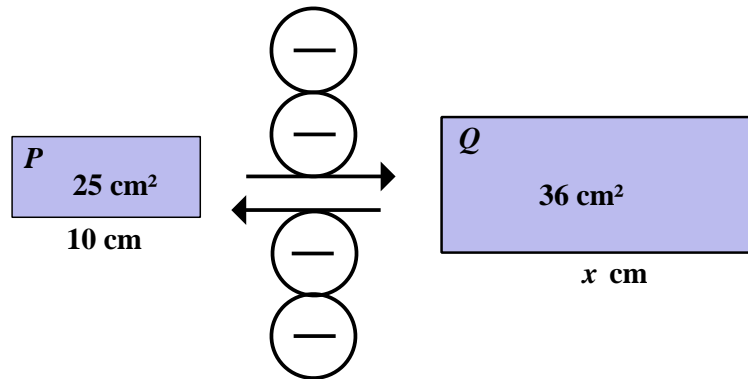
Question 1



Rectangles P and Q are similar.
Find the length marked a

[3 marks]

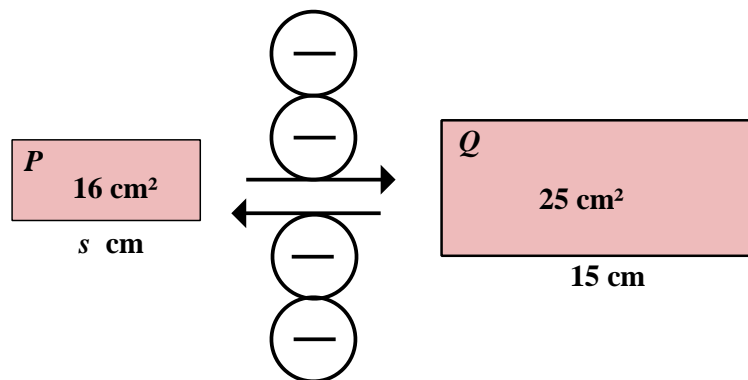
Question 2



Rectangles P and Q are similar.
Find the length marked x

[3 marks]

Question 3



Rectangles P and Q are similar.
Find the length marked s

[3 marks]

Question 4

Two similar shapes have an area scale factor of $\left(\frac{50}{32}\right)$.

Oliver has forgotten his calculator (again!) and cannot see how to square root the 50 or the 32 in order to get the length scale factor between the two similar shapes. However, there is a way of getting the length scale factor without using a calculator. Explain how.

[3 marks]

2.6 Exercise

Marks Available : 34

Question 1

(i)

$$\left(\frac{3}{8}\right)^2 = \underline{\hspace{2cm}}$$

(ii)

$$\left(\frac{5}{4}\right)^2 = \underline{\hspace{2cm}}$$

(iii)

$$\left(\frac{7}{9}\right)^2 = \underline{\hspace{2cm}}$$

(iv)

$$\sqrt{\frac{36}{49}} = \underline{\hspace{2cm}}$$

(v)

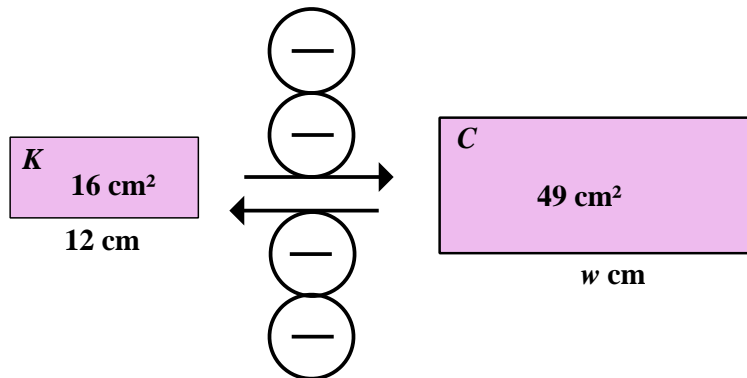
$$\sqrt{\frac{100}{9}} = \underline{\hspace{2cm}}$$

(vi)

$$\left(\frac{64}{81}\right)^{0.5} = \underline{\hspace{2cm}}$$

[6 marks]

Question 2



Rectangles K and C are similar.

Find the length marked w

[3 marks]

Question 3

Two similar shapes have a *length scale factor* of $\frac{6}{5}$

What is their *area scale factor* ?

[1 mark]

Question 4

Two similar shapes have an *area scale factor* of $\frac{9}{16}$

What is their *length scale factor* ?

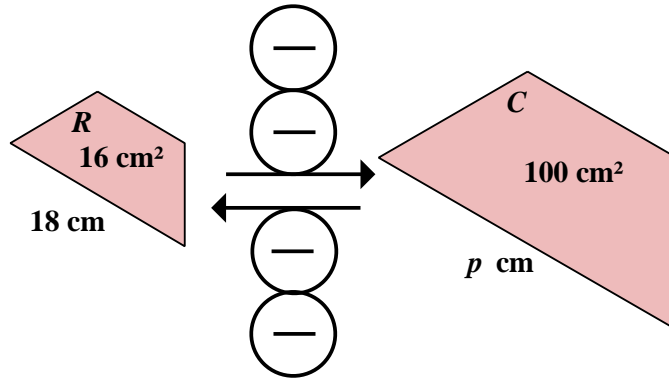
[1 mark]

Question 5

Two similar shapes have a *length scale factor* of $\frac{4}{9}$
What is their *area scale factor* ?

[1 mark]

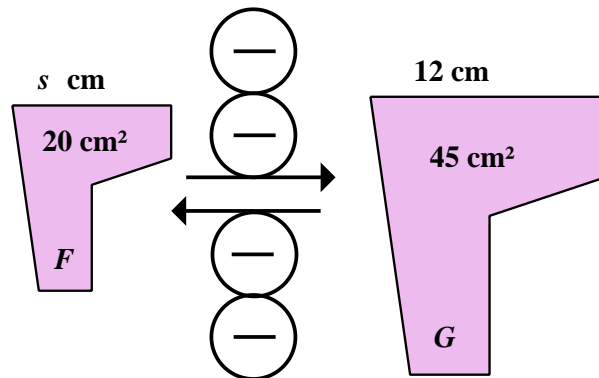
Question 6



The shapes *R* and *C* are similar.
Find the length marked *p*

[3 marks]

Question 7



The shapes *F* and *G* are similar.

- (i) What is the area scale factor (larger than 1) of the similarity ?
Give your answer as a fraction "cancelled down".

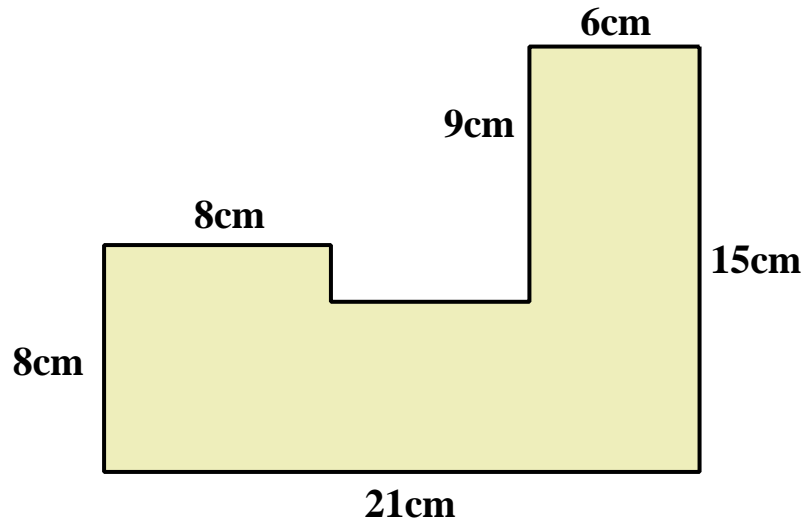
[1 mark]

- (ii) Find the length marked *s*

[2 marks]

Question 8

(i) Work out the shaded area of the shape shown below.



[3 marks]

(ii) In a larger similar shape, corresponding to the side of length 21 cm is a side of length 24 cm.

(a) What is the *length scale factor* of the similarity ?

[1 mark]

(b) What is the *area scale factor* of the similarity ?

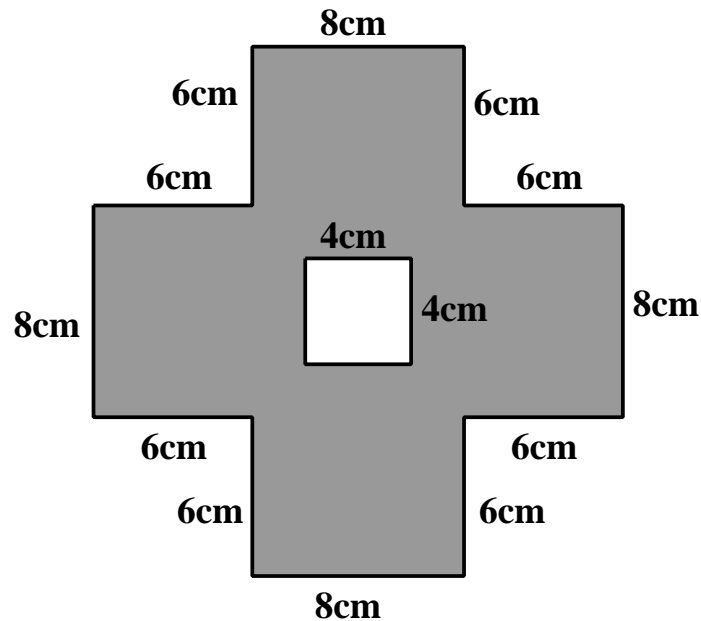
[1 mark]

(c) Use your part (i) answer and your part (ii) (b) answer to work out the area of the larger similar shape.

[2 marks]

Question 9

- (i) (a) Work out the area of the white square in the shape below. [1 mark]
- (b) Work out the area shown shaded below. [2 marks]



- (ii) A similar, larger shape is to be drawn, using a *length scale factor* of $\frac{5}{4}$.
- (a) What will be the length, in the similar shape, of the 8cm sides ? [1 mark]
- (b) What will be the length, in the similar shape, of the 6cm sides ? [1 mark]
- (c) What will be the length, in the similar shape of the 2cm sides ? [1 mark]
- (d) What will be are *area scale factor* of the similarity ? [1 mark]
- (e) Use your part (i) (b) answer and your part (ii) (d) answer to work out the area of the larger, similar shape. [2 marks]