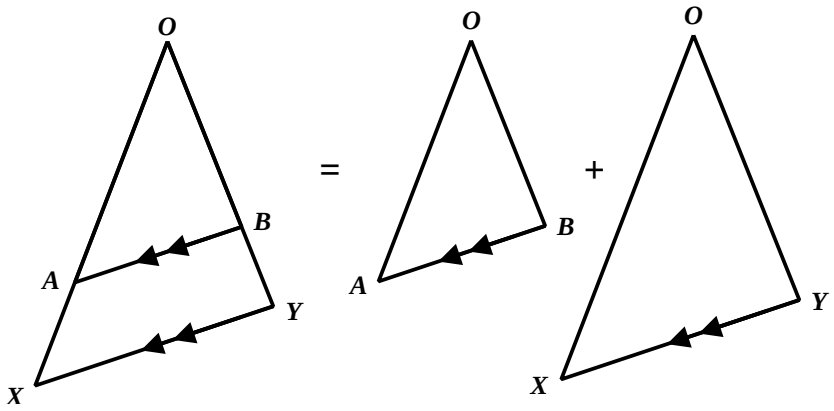
Lesson 7

GCSE Mathematics
Ratio and Similarity

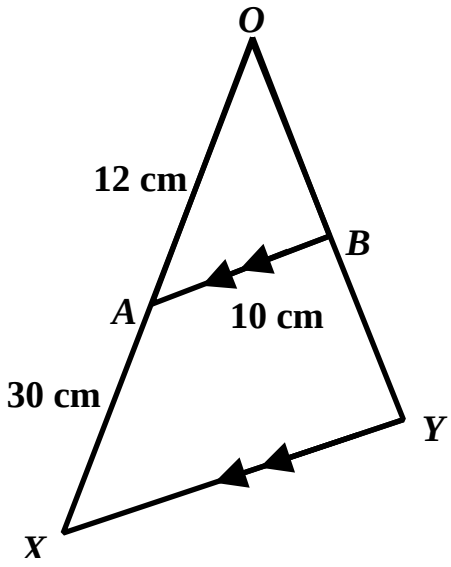
7.1 Superimposed Triangles

Similar triangles are often encountered in situations where one triangle is on top of the other. In the following diagram, the left hand figure can be thought of as being made up from the two triangles ΔOAB and ΔOXY placed one upon the other.



The two separated triangles are ***similar*** because AB is parallel to XY.
So although they are of different sizes, they have the same shape.
In fact ΔOXY is an enlargement of ΔOAB.

Example
Let OA = 12 cm, AX = 30 cm and AB = 10 cm.
AB is parallel with XY.
Find the length of XY.

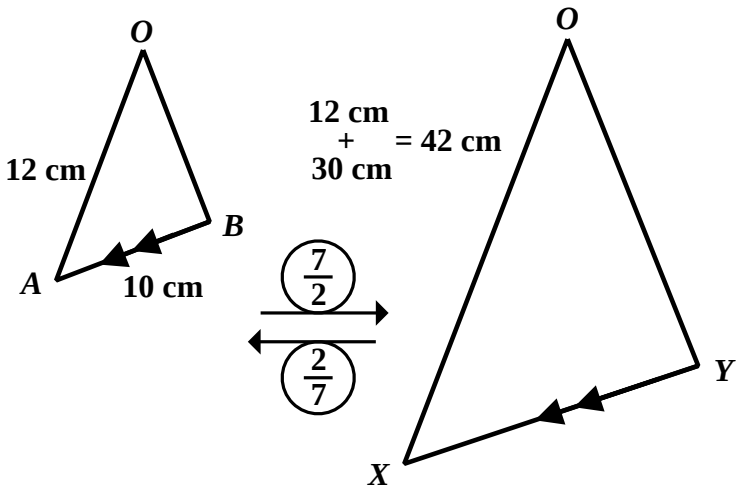


The trap for the unaware in this question is to think that the length scale factor between the two similar triangles is $\frac{30}{12}$ or $\frac{6}{5}$. It isn't !
This is because the length of the larger triangle's longest side is not 30 cm.

What is the length of the larger triangles longest side ?

[1 mark]

Having turned the page, I hope you first answered, 42 cm.



$$length\ scale\ factor\ = \frac{42}{12} = \frac{21}{6} = \frac{7}{2}$$

$$\therefore\ length\ XY\ = 10 \times \frac{7}{2}$$

$$= 35\ \text{cm}$$

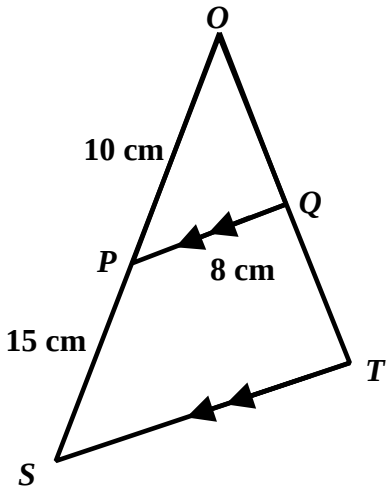
[3 marks]

7.2 Exercise

Marks Available : 40

Question 1

Let OP = 10 cm, PS = 15 cm and PQ = 8 cm.
PQ is parallel with ST.



(i) Draw the two superimposed triangles separately in the space above.

[2 marks]

(ii) Find the length scale factor, greater than 1, between the two triangles.

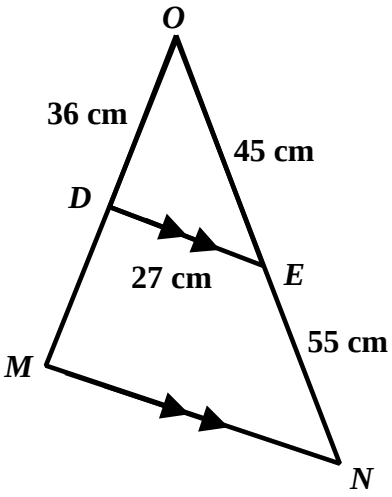
[2 marks]

(iii) Find the length of ST.

[2 marks]

Question 2

Let OD = 36 cm, OE = 45 cm, DE = 27 cm and EN = 55 cm.
DE is parallel with MN.



(i) Draw the two superimposed triangles separately in the space above.

[2 marks]

(ii) Find the length scale factor, greater than 1, between the two triangles.

[2 marks]

(iii) Find the length of MN.

[1 mark]

(iv) Find the length of OM.

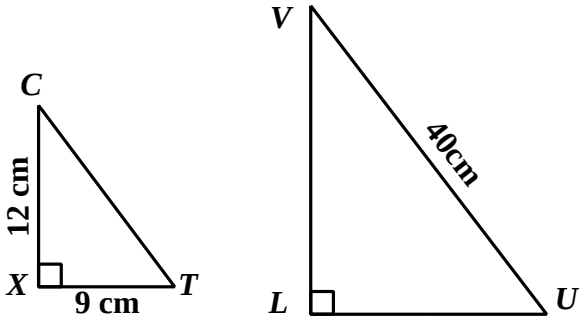
[1 mark]

(v) Find the length of DM.

[1 mark]

Question 3

$\triangle XTC$ and $\triangle LUV$ are similar with length LU shorter than length LV.



(i) Use the theorem of Pythagoras to find the length of side TC in $\triangle XTC$.

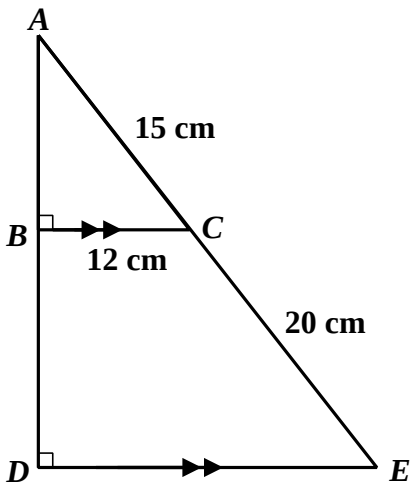
[2 marks]

(ii) Hence find lengths LU and LV in $\triangle LUV$.

[2 marks]

Question 4

Let AC = 15 cm, BC = 12 cm, and CE = 20.
BC is parallel with DE.



(i) Draw the two superimposed triangles separately in the space above.

[2 marks]

(ii) Use the Theorem of Pythagoras to calculate length AB.

Think : If your answer is more than the 15 cm hypotenuse you've done something wrong !

[2 marks]

(iii) What is the length scale factor, greater than 1, between the two triangles ?

[2 marks]

(iv) What is the area scale factor, greater than 1, between the two triangles ?

[1 mark]

(v) What is the area of ΔABC ?

[1 mark]

(vi) What is the area of ΔADE ?

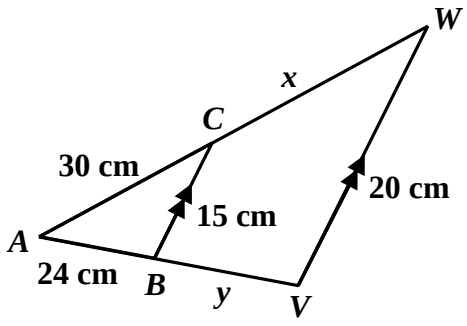
[1 mark]

(vii) What is the area of the trapezium $BDEC$?

[2 marks]

Question 5

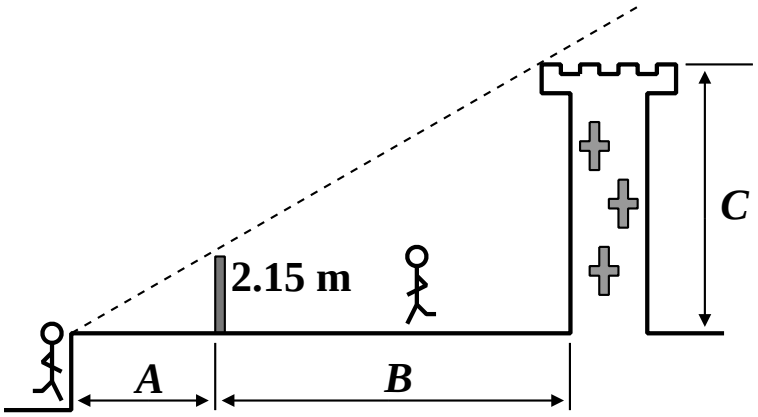
Find x and y.



[6 marks]

Question 6

Sid and Fred are having an argument.
Sid says that the castle tower is more than 100m high.
Fred says it's less than 100m high.



The castle tower is a ruin and locked. So Sid and Fred can't get to the top of it to dangle a rope to the ground and simply measure the length of the rope.

With a £250 bet riding on the outcome they offer to pay a well known mathematician, (that's you by the way!) £50 to answer the question, "Is the Tower more or less than 100 metres high ?"

You advise them to place a stick in the flat, level ground such that when viewed from a ditch, the top of the stick and the top of the tower line up as shown in the diagram.

Everything is now measured carefully with you, Sid and Fred agreeing on the measurements taken. These are;

Top of stick is precisely 2.15 metres above the ground.
Distance *A* is 2.5 metres exactly.
Distance *B* is 115.5 metres without any doubt what-so-ever.

So, is the Tower more or less than 100 metres high ?
You will need to provide clear working otherwise Sid and Fred will continue to argue.

[6 marks]