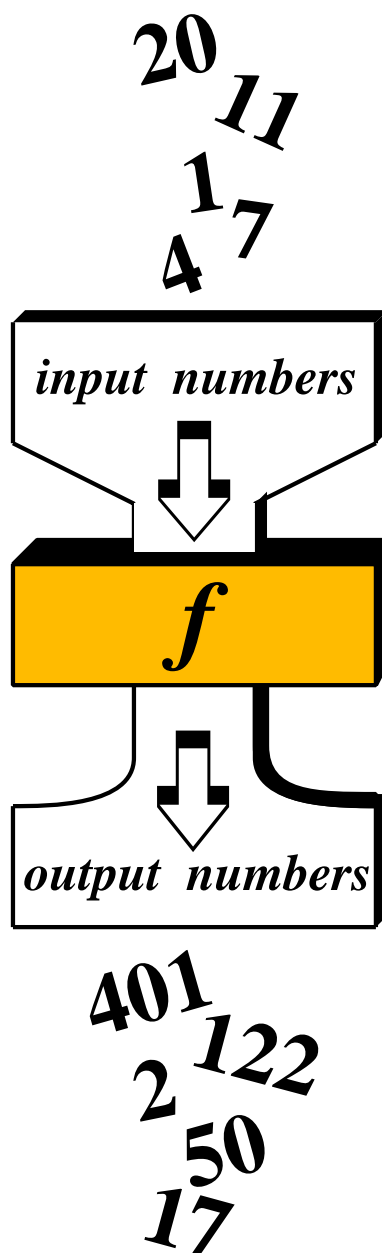


GCSE Mathematics

# FUNCTIONS

## I



# Functions I

## Lesson 1

## GCSE Mathematics Functions I

### 1.1 What is a function ?

At a simple level, a function is a mathematical rule that “does things to numbers”.  
If it is helpful, a function can be given a name.

Here is a description of the function called  $f$ .

$$f(x) = 1 + \sqrt{x}, \quad x \in \mathbb{Z}, \quad x \geq 0$$

Now, suppose I have the number 25.

What does function  $f$  do to the number 25 ?

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(1\).mp4](http://www.NumberWonder.co.uk/Video/v9002(1).mp4)



[ 2 marks ]

### 1.2 For You To Do

Consider the function,  $f(x) = x^2 + 1$ ,  $x \in \mathbb{R}$

Calculate,

(i)  $f(7)$

(ii)  $f(1)$

(iii)  $f(11)$

(iv)  $f(0)$

(v)  $f(5)$

(vi)  $f(-5)$

(vii)  $f(-6)$

(viii)  $f(\sqrt{3})$

[ 8 marks ]

Now turn over the page to see if you got these questions correct.

### 1.3 Answers to For You To Do

In words the function  $f(x) = x^2 + 1$  takes the number it is given and “squares it and adds one”

When given the number 7, this function it will square 7 and add 1

$$\begin{aligned}f(x) &= x^2 + 1 \\f(7) &= 7^2 + 1 \\&= 49 + 1 \\&= 50\end{aligned}$$

Repeating this for all of the questions gives the following answers,

- (i)  $f(7) = 50$       (ii)  $f(1) = 2$   
(iii)  $f(11) = 122$       (iv)  $f(0) = 1$   
(v)  $f(5) = 26$       (vi)  $f(-5) = 26$       because  $(-5)(-5) = 25$   
(vii)  $f(-6) = 37$       (viii)  $f(\sqrt{3}) = 4$       because  $(\sqrt{3})(\sqrt{3}) = 3$

[ 8 marks ]

Finally, notice the domain,  $x \in \mathbb{R}$

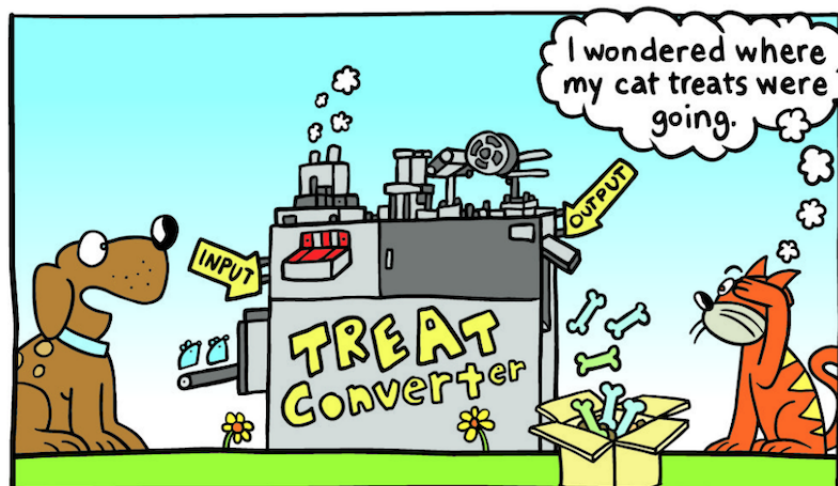
Any Real Number may be used as input into the “square and adds one” function.

The Real Numbers are “all the numbers you officially know about”.

This includes Integers,  $\mathbb{Z}$ , Rational Numbers,  $\mathbb{Q}$ , and Irrational Numbers.

There are lots of numbers that you “don't officially know about” !

Further A-Level Mathematicians, for example, know of Complex Numbers,  $\mathbb{C}$ .



**“It is my treat-converter function machine. However many cat treats I input, the machine outputs TWICE that many dog biscuits. Isn't that cool?”**

It's not just numbers that can go into functions !  
(For example, later on we'll input algebraic expressions)

## 1.4 Exercise

Marks Available : 40

### Question 1

Function  $f$  is described as,

$$f(x) = 4x + 13, \quad x \in \mathbb{Z}$$

Write down the value of,

(i)  $f(1)$                       (ii)  $f(2)$                       (iii)  $f(3)$

(iv)  $f(0)$                       (v)  $f(10)$                       (vi)  $f(-2)$

[ 3 marks ]

What does the domain,  $x \in \mathbb{Z}$ , tell you about this function ?



[ 1 mark ]

### Question 2

Function  $g$  is described as,

$$g(x) = \frac{x^2}{2} \quad x \in \mathbb{Z}$$

Write down the value of,

(i)  $g(1)$                       (ii)  $g(-1)$                       (iii)  $g(4)$

(iv)  $g(10)$                       (v)  $g(0)$                       (vi)  $g(-3)$

[ 3 marks ]

The range of this function, the numbers that can occur as answers, are all non negative. Why this is ?



[ 1 mark ]

### Question 3

Function  $h$  is the function described by,  $h(x) = 7 + \sqrt{x}$ ,  $x \in \mathbb{R}$ ,  $x \geq 0$

Write down the value of,

(i)  $h(100)$                       (ii)  $h(4)$                       (iii)  $h(81)$

(iv)  $h(0)$                       (v)  $h(400)$                       (vi)  $h(121)$

[ 3 marks ]

Explain why the restriction is placed upon the domain that only numbers greater than or equal to zero can be allowed into this function.



[ 1 mark ]

### Question 4

Given that,  $m(x) = 14 + 3x$ ,  $x \in \mathbb{Z}$ , write down the value of,

(i)  $m(1)$                       (ii)  $m(-1)$                       (iii)  $m(6)$

(iv)  $m(9)$                       (v)  $m(-3)$                       (vi)  $m(-5)$

[ 3 marks ]

### Question 5

Given that,  $n(x) = 10 - 2x$ ,  $x \in \mathbb{R}$ , write down the value of,

(i)  $n(4)$                       (ii)  $n(0)$                       (iii)  $n(-2)$

(iv)  $n(11)$                       (v)  $n(-8)$                       (vi)  $n\left(\frac{1}{2}\right)$

[ 3 marks ]

### Question 6

Consider the function,  $d(x) = \frac{6}{x}$ ,  $x \in \mathbb{R}$ ,  $x \neq 0$

Write down the value of,

- (i)  $d(3)$                       (ii)  $d(6)$                       (iii)  $d(12)$
- (iv)  $d(-2)$                       (v)  $d(0.1)$                       (vi)  $d(-0.1)$

[ 3 marks ]

Explain the restriction  $x \neq 0$  on the domain.

In other words, why is the number zero is not allowed into this function ?



[ 1 mark ]

### Question 7

The function  $e$  is given by,  $e(x) = \sqrt{4x + 1}$ ,  $x \in \mathbb{R}$ ,  $x \geq -0.25$

Write down the value of,

- (i)  $e(0)$                       (ii)  $e(2)$                       (iii)  $e(6)$
- (iv)  $e(12)$                       (v)  $e\left(\frac{3}{4}\right)$                       (vi)  $e\left(-\frac{1}{4}\right)$

[ 3 marks ]

### Question 8

The function  $k$  is given by,  $k(x) = 4x^2 + 3$ ,  $x \in \mathbb{R}$

Write down the value of,

- (i)  $k(1)$                       (ii)  $k(-1)$                       (iii)  $k(2)$
- (iv)  $k(-2)$                       (v)  $k(5)$                       (iv)  $k(-5)$

[ 3 marks ]

**Question 9**

The quadratic function  $A$  is,  $A(x) = 2x^2 + 3x - 1$ ,  $x \in \mathbb{R}$

Write down the value of,

( i )  $A(1)$                       ( ii )  $A(0)$                       ( iii )  $A(5)$

( iv )  $A(10)$                       ( v )  $A(100)$                       ( iv )  $A(-1)$

[ 3 marks ]

**Question 10**

The function  $H$  has the definition,  $H(x) = (2x + 5)(3x + 4)$ ,  $x \in \mathbb{Z}$

Write down the value of,

( i )  $H(1)$                       ( ii )  $H(0)$                       ( iii )  $H(-2)$

( iv )  $H(-1)$                       ( v )  $H(2)$                       ( vi )  $H(-3)$

[ 3 marks ]

**Question 11**

This question is about a mystery function,  $M$ .

We can drop numbers into  $M$ , and see what it does to those numbers.

If I drop the numbers 0, 1, 2, 3, 4 and 5 into  $M$  then 1, 4, 7, 10, 13 and 16 come out.

In other words     $M(0) = 1$   
                           $M(1) = 4$   
                           $M(2) = 7$   
                           $M(3) = 10$   
                           $M(4) = 13$   
                          and     $M(5) = 16$

Suggest a rule that  $M$  is applying.

*It must turn all five input numbers into the five output numbers.*

*If your rule only works on one or two numbers, it's no good !*

[ 3 marks ]

### Question 12

Here is another “guess the rule” question.

$$N(0) = -1$$

$$N(1) = 0$$

$$N(2) = 3$$

$$N(3) = 8$$

$$N(4) = 15$$

and  $N(5) = 24$

Suggest a function that  $N$  could be.

[ 3 marks ]