4.1 What was the Input ?

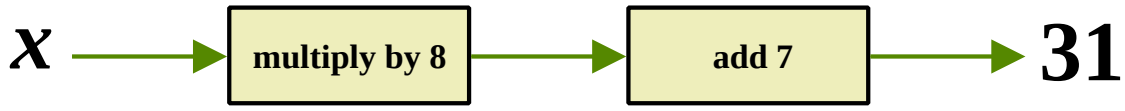
In the work on functions so far, the questions have all revolved around the inputs of either a number or an algebraic expression into the function and deducing what would, in consequence, be the output.
With some functions, but not all, it's possible to reverse this process.
In other words, given what has come out, which could be a number or some algebra, what was it that went in ?

4.2 The Game of Opposites

Here is how such a question might be asked;

"If $f(x) = 8x + 7$, find x such that $f(x) = 31$. $(x \in \mathbb{R})$"

A flow diagram is an excellent way of understanding exactly what this question is asking. As so often in maths, what is not known, in this case the input, is called x.
The flow from left to right shows that the x is first multiplied by 8 and then 7 is added on, giving the number 31 as an output.



To work out x, instead of flowing left to right, the flow is reversed and goes from right to left. It's now a "Game of Opposites" !
The opposite of adding 7 is subtracting 7.
The opposite of multiplying by 8 is dividing by 8.

Here is the opposites flow diagram.

Decide, in your mind what the value of x must be.

A more mathematical way to do this question is as follows,

$$8x + 7 = 31$$

$$8x = 31 - 7 \qquad \text{Subtract 7 from both sides}$$

$$8x = 24$$

$$x = \frac{24}{8} \qquad \text{Divide both sides by 8}$$

$$x = 3$$

Here is a second example. Try it yourself, then check with my answer over the page.
If $f(x) = 6x + 11$, find x such that $f(x) = 53$. $(x \in \mathbb{R})$

$$\text{Solve}, \quad 6x + 11 = 53$$
$$6x = 53 - 11$$
$$6x = 42$$
$$x = \frac{42}{6}$$
$$x = 7$$

4.3 Exercise

Try doing some questions using a flow diagram, and some using just algebra. The algebraic method always works but often the flow diagram provides is a much quicker and easier short cut to the answer.

Marks Available: 78

Question 1

If $f(x) = 4x + 13$, find x such that $f(x) = 41$ $(x \in \mathbb{R})$

[3 marks]

Question 2

If $g(x) = 18 + 5x$, find x such that $g(x) = 103$ $(x \in \mathbb{R})$

[3 marks]

Question 3

If $h(x) = 3(8x - 3)$, find x such that $h(x) = 135$ $(x \in \mathbb{R})$

[3 marks]

Question 4

If $k(x) = \frac{5x}{4} + 7$, find x such that $k(x) = 37$ $(x \in \mathbb{R})$

[3 marks]

Question 5

If $m(x) = \frac{3x}{5} - 1$, find x such that $m(x) = 32$ $(x \in \mathbb{R})$

[3 marks]

Question 6

If $n(x) = 68 - 7x$, find x such that $n(x) = 26$ $(x \in \mathbb{R})$

[3 marks]

Question 7

If $p(x) = 6(2x + 3)$, find x such that $p(x) = 48$ $(x \in \mathbb{R})$

[3 marks]

Question 8

If $q(x) = 6(13 - x)$, find x such that $q(x) = 72$ $(x \in \mathbb{R})$

[3 marks]

Question 9

If $r(x) = \frac{x + 17}{3}$, find x such that $r(x) = 40$ $(x \in \mathbb{R})$

[3 marks]

Question 10

If $s(x) = \frac{3x + 2}{5}$, find x such that $s(x) = 10$ $(x \in \mathbb{R})$

[3 marks]

Question 11

If $t(x) = 5x^3$, find x such that $t(x) = 320$ $(x \in \mathbb{R})$

[3 marks]

Question 12

If $u(x) = \frac{21 - x}{4}$, find x such that $u(x) = 6$ $(x \in \mathbb{R})$

[3 marks]

Question 13

If $v(x) = 14\sqrt{x} + 111$, $x \in \mathbb{R}$, $x \geqslant 0$, find x such that $v(x) = 181$

[3 marks]

Question 14

If $w(x) = \dfrac{3x - 5}{x - 4}$, $x \in \mathbb{R}$, $x \neq 4$, find x such that $w(x) = 4$

[3 marks]

Question 15

If $a(x) = \dfrac{\sqrt{x} + 7}{2}$, $x \in \mathbb{R}, x \geqslant 0$, find x such that $a(x) = 16$

[3 marks]

Question 16

If $b(x) = \frac{x + 2}{3} + 9$, find x such that $b(x) = 14$ $\quad (x \in \mathbb{R})$

[3 marks]

Question 17

If $c(x) = \frac{105}{x}$, $x \in \mathbb{R}$, $x \neq 0$, find x such that $c(x) = 7$

[3 marks]

Question 18

If $d(x) = \frac{5x + 1}{x + 1}$, $x \in \mathbb{R}$, $x \neq -1$, find x such that $d(x) = 4$

[3 marks]

Question 19

If $e(x) = \sqrt[3]{x} + 7$, find x such that $e(x) = 11$ $\quad (x \in \mathbb{R})$

[3 marks]

Question 20

If $f(x) = \dfrac{5x - 4}{x + 1}$, $x \in \mathbb{R}$, $x \neq -1$, find x such that $f(x) = 8$

[3 marks]

Question 21

If $g(x) = \dfrac{2}{x} + \dfrac{3}{x}$, $x \in \mathbb{R}$, $x \neq 0$, fund x such that $g(x) = 20$

[3 marks]

Question 22

If $h(x) = \sqrt{13 + 3x}$, $x \in \mathbb{R}$, $x \geqslant -\frac{13}{3}$, find x such that $h(x) = 7$

[3 marks]

Question 23

If $k(x) = 29 - 24\sqrt{x}$, $x \in \mathbb{R}$, $x \geqslant 0$, find x such that $k(x) = 17$

[3 marks]

Question 24

If $p(x) = x^2 + x$, find all x such that $p(x) = 1$ $\quad (x \in \mathbb{R})$

Note: The equation $ax^2 + bx + c = 0$, where a, b and c are constants

has the solutions given by, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

[3 marks]

Question 25

If $m(x) = \dfrac{3}{x} + \dfrac{5}{2x}$, $x \in \mathbb{R}$, $x \neq 0$, find x such that $m(x) = 33$

[3 marks]

Question 26

If $n(x) = \frac{1}{3x} + \frac{3}{5x}, \quad x \in \mathbb{R}, \quad x \neq 0,$ find x such that $n(x) = 7$

[3 marks]