

Lesson 5

GCSE Mathematics Functions I

5.1 Composite Functions (Algebra Questions)

Two functions, f and g given by,

$$f(x) = 5x - 2 \quad x \in \mathbb{R}$$

$$g(x) = 4x + 1 \quad x \in \mathbb{R}$$

What is $fg(2)$?



[2 marks]

In this Lesson the function inputs will be chunks of algebra rather than just numbers.

Work out $fg(2z)$



[2 marks]

Now try to work out $fg(3z + 1)$



[2 marks]

My answer is over the page.
Turn over to see if you concur.

$$\begin{aligned}
fg(3z + 1) &= f(4(3z + 1) + 1) && \text{because } g(x) = 4x + 1 \\
&= f(12z + 4 + 1) \\
&= f(12z + 5) \\
&= 5(12z + 5) - 2 && \text{because } f(x) = 5x - 2 \\
&= 60z + 25 - 2 \\
&= 60z + 23
\end{aligned}$$

[2 marks]

5.2 Exercise

Marks Available: 80

Question 1

Let p and q be the functions;

$$p(x) = 3x + 7 \quad x \in \mathbb{R}$$

$$q(x) = 10x + 9 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $p q(0)$

(ii) $p q(1)$

(iii) $p q(5z)$

(iv) $p q(2z + 3)$

(v) $q q(-1)$

(vi) $p p p(-3)$

[6 marks]

Question 2

Let h and d be the functions;

$$h(x) = 3x + 7 \quad x \in \mathbb{R}$$

$$d(x) = 5x + 9 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $hd(4)$

(ii) $hd(-1)$

(iii) $hd(4z)$

(iv) $hd(3z+3)$

[4 marks]

Question 3

Let f and g be the functions;

$$f(x) = 6x - 5 \quad x \in \mathbb{R}$$

$$g(x) = 2x + 11 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $gf(0)$

(ii) $gf(2)$

(iii) $gf(3z)$

(iv) $gf(5z+1)$

[4 marks]

Question 4

$$f(x) = x + 3 \quad x \in \mathbb{R}$$

$$g(x) = 4x \quad x \in \mathbb{R}$$

Find $fg(x)$

[2 marks]

Question 5

$$f(x) = 3x + 2 \quad x \in \mathbb{R}$$

$$g(x) = 5x - 4 \quad x \in \mathbb{R}$$

Find an expression for $gf(x)$ that does not contain any brackets.

[2 marks]

Question 6

$$f(x) = 7x - 5 \quad x \in \mathbb{R}$$

$$g(x) = 10 - x \quad x \in \mathbb{R}$$

Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 7

$$f(x) = x^2 + 5 \quad x \in \mathbb{R}$$

$$g(x) = 2x - 1 \quad x \in \mathbb{R}$$

Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 8

$$f(x) = 7x - 6 \quad x \in \mathbb{R}$$

$$g(x) = \frac{3}{x + 4} \quad x \in \mathbb{R}, x \neq -4$$

- (i) Find an expression for $f g(x)$ that does not contain any brackets.

[2 marks]

- (ii) Find an expression for $g f(x)$ that does not contain any brackets.

[2 marks]

Question 9

$$f(x) = 5x + 2 \quad x \in \mathbb{R}$$

$$g(x) = 4x - 9 \quad x \in \mathbb{R}$$

- (i) Calculate $f g(3)$

[2 marks]

- (ii) Find an expression for $f g(x)$ that does not contain any brackets.

[2 marks]

- (iii) Now use your part (ii) formula to calculate $f g(3)$ again.
Check it agrees with your answer to part (i) !

[1 mark]

Question 10

$$f(x) = x^2 - 6 \quad x \in \mathbb{R}$$

$$g(x) = 3x + 1 \quad x \in \mathbb{R}$$

(i) Show carefully that $fg(x) = 9x^2 + 6x - 5$

[2 marks]

(ii) Hence solve the equation $fg(x) = 43$

[2 marks]

Question 11

$$f(x) = 2x^2 + 3x - 4 \quad x \in \mathbb{R}$$

$$g(x) = x - 5 \quad x \in \mathbb{R}$$

(i) Show carefully that $fg(x) = 2x^2 - 17x + 31$

[2 marks]

(ii) Hence solve the equation $fg(x) = 10 - 4x$

[2 marks]

Question 12

Let u and v be the functions;

$$u(x) = x^2 - 5 \quad x \in \mathbb{R}$$

$$v(x) = 5x + 2 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $v u(3)$

(ii) $v u(2)$

(iii) $v u(5z)$

(iv) $v u(2z + 1)$ **HINT : FOIL**

[8 marks]

Question 13

Let m and n be the functions,

$$m(x) = 9x - 5 \quad x \in \mathbb{R}$$

$$n(x) = \sqrt{x - 7} \quad x \in \mathbb{R}, x \geq 7$$

Evaluate each of the following,

(i) $m n(8)$

(ii) $m n(56)$

(iii) $m n(z^2 + 7)$

(iv) $m n(4z^2 + 7)$

[8 marks]

Question 14

$$f(x) = 3x + 7 \quad x \in \mathbb{R}$$

$$g(x) = \frac{5}{x - 2} \quad x \in \mathbb{R}, x \neq 2$$

- (i) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

- (ii) Find an expression for $gf(x)$ that does not contain any brackets.

[2 marks]

Question 15

$$f(x) = 4x + 5 \quad x \in \mathbb{R}$$

$$g(x) = 3x - 7 \quad x \in \mathbb{R}$$

- (i) Calculate $fg(2)$

[2 marks]

- (ii) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

- (iii) Now use your part (ii) formula to calculate $fg(2)$ again.
Check it agrees with your answer to part (i) !

[1 mark]

Question 16

$$f(x) = x^2 + 8 \quad x \in \mathbb{R}$$

$$g(x) = 2x - 5 \quad x \in \mathbb{R}$$

- (i) Show carefully that $fg(x) = 4x^2 - 20x + 33$

[2 marks]

- (ii) Hence solve the equation $fg(x) = 17$

[2 marks]

Question 17

$$f(x) = 3x^2 - 2x + 5 \quad x \in \mathbb{R}$$

$$g(x) = x + 4 \quad x \in \mathbb{R}$$

- (i) Show carefully that $fg(x) = 3x^2 + 22x + 45$

[2 marks]

- (ii) Hence solve the equation $fg(x) = 27x + 43$

[2 marks]

Question 18

Let f and g be the functions;

$$f(x) = 7 - 6x \quad x \in \mathbb{R}$$

$$g(x) = 5x + 7 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $fg(-1)$

(ii) $fg(-0.2)$

(iii) $fg(4z + 1)$

(iv) $fg(1 - z^2)$

[8 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

(Questions 4 to 11, and 14 to 17 JCA)