9.1 "Worst Case" Inverse Functions

In the GCSE examination there is a particularly awkward class of function to which the inverse may be requested.

The function referred to is known as a linear rational function and is of the form,

$$f(x) = \frac{ax + b}{cx + d} \qquad a,\ b,\ c,\ d \in \mathbb{Z}, \quad x \in \mathbb{R}, \quad x \neq -\frac{d}{c}$$

9.2 Example

Find the inverse function of the following function,

$$f(x) = \frac{5x - 1}{2x + 3} \qquad x \in \mathbb{R}, \ x \neq -\frac{3}{2}$$

Teaching Video : http://www.NumberWonder.co.uk/Video/v9002(9).mp4

[4 marks]

Check if your answer could be correct by working out $f(7)$ and then inserting the answer into your proposed inverse function.

[1 mark]

9.3 Exercise

Question 1

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(**i**) $b(x) = \dfrac{7 - x}{x}, \quad x \neq 0$

[4 marks]

(**ii**) $c(x) = \dfrac{5 - 3x}{x}, \quad x \neq 0$

[4 marks]

Question 2

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $d(x) = \dfrac{x+3}{x+2}, \quad x \neq -2$

[4 marks]

(ii) $e(x) = \dfrac{x+1}{x-2}, \quad x \neq 2$

[4 marks]

Question 3

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(**i**) $f(x) = \dfrac{1}{2x} + \dfrac{1}{3x}, \quad x \neq 0$

[4 marks]

(**ii**) $g(x) = \dfrac{3}{5x} + \dfrac{1}{4x} + 2. \quad x \neq 0$

[4 marks]

Question 4

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $h(x) = \dfrac{x^2 + 5x + 6}{x^2 + 6x + 8}, \qquad x \ne -4, -2$

HINT: separately factorise the numerator and the denominator

[4 marks]

(ii) $k(x) = \dfrac{x^2 - 4}{x^2 - 4x + 4}, \quad x \ne 2$

[4 marks]