

## Lesson 9

## GCSE Mathematics Functions I

### 9.1 “Worst Case” Inverse Functions

In the GCSE examination there is a particularly awkward class of function to which the inverse may be requested.

The function referred to is known as a linear rational function and is of the form,

$$f(x) = \frac{ax + b}{cx + d} \quad a, b, c, d \in \mathbb{Z}, \quad x \in \mathbb{R}, \quad x \neq -\frac{d}{c}$$

### 9.2 Example

Find the inverse function of the following function,

$$f(x) = \frac{5x - 1}{2x + 3} \quad x \in \mathbb{R}, \quad x \neq -\frac{3}{2}$$

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(9\).mp4](http://www.NumberWonder.co.uk/Video/v9002(9).mp4)



[ 4 marks ]

Check if your answer could be correct by working out  $f(7)$  and then inserting the answer into your proposed inverse function.

[ 1 mark ]

### 9.3 Exercise

Marks Available: 32

#### Question 1

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers,  $x \in \mathbb{R}$

(i)  $b(x) = \frac{7 - x}{x}, \quad x \neq 0$

[ 4 marks ]

(ii)  $c(x) = \frac{5 - 3x}{x}, \quad x \neq 0$

[ 4 marks ]

**Question 2**

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers,  $x \in \mathbb{R}$

(i)  $d(x) = \frac{x + 3}{x + 2}, \quad x \neq -2$

[ 4 marks ]

(ii)  $e(x) = \frac{x + 1}{x - 2}, \quad x \neq 2$

[ 4 marks ]

**Question 3**

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers,  $x \in \mathbb{R}$

(i)  $f(x) = \frac{1}{2x} + \frac{1}{3x}, \quad x \neq 0$

[ 4 marks ]

(ii)  $g(x) = \frac{3}{5x} + \frac{1}{4x} + 2, \quad x \neq 0$

[ 4 marks ]

#### Question 4

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers,  $x \in \mathbb{R}$

(i)  $h(x) = \frac{x^2 + 5x + 6}{x^2 + 6x + 8}, \quad x \neq -4, -2$

HINT: separately factorise the numerator and the denominator

[ 4 marks ]

(ii)  $k(x) = \frac{x^2 - 4}{x^2 - 4x + 4}, \quad x \neq 2$

[ 4 marks ]

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