Lesson 3

3.1 The Elements of a Set

In Set Theory, great importance is attached to whether an object is in a set or not.
The objects that are in a set are called the elements of that set.
Another word for "element" is "member"
In Set Theory the symbol $\in$ means "is an element of"
Think of : $\in$l$\in$m$\in$nt

Consider the set of people's names beginning with the letter L

$$L = \{ Libby, \ Louis, Lyla, \ Livi, \ \ldots \}$$

I can write

$$Lyla \in L$$

This reads as

"*Lyla* is an element of set *L*"

In other words,

"*Lyla* is a member of the club of people
who have a name begin with the letter L"

I can also write

$$Archie \notin L$$

This reads as

"*Archie* is NOT an element of set *L*"

In other words,

"*Archie* is a NOT member of the club of people
who have a name begin with the letter L"

Here are four statements and you now have to say if they are TRUE or FALSE

(i) $Livi \in L$

(ii) $Livi \notin L$

(iii) $Alex \notin L$

(iv) $Alex \in L$

[4 marks]

As mathematicians we're interested in doing this sort of thing with numbers.
The answer is that the first and third are true and the other two false.
Did you get it correct ?

3.2 Exercise

Marks Available : 100

Question 1

Let P = { Prime Numbers }

i.e. P = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, ... }

For each of the following, decide if the given statement is TRUE or FALSE

(i) $7 \in P$

(ii) $9 \notin P$

(iii) $27 \in P$

(iv) $1 \notin P$

(v) $49 \notin P$

(vi) $17 \in P$

(vii) $2 \notin P$

(viii) $91 \in P$

(ix) $369 \notin P$

(x) $\pi \in P$

[10 marks]

Question 2

Let S = { Square Numbers }

i.e. S = { 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ... }

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $2 \in S$

(ii) $15 \notin S$

(iii) $36 \in S$

(iv) $91 \notin S$

(v) $100 \notin S$

(vi) $101 \in S$

(vii) $1 \notin S$

(viii) $400 \in S$

(ix) $\sqrt{16} \in S$

(x) $\pi \in S$

[10 marks]

The next question is going to ask you whether certain numbers are in the following sequence or not
3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, 79, 83, ...

Each term is four more than the term before.

Here is a clever "Matched Columns" technique that focuses on the right hand end of the numbers. By starting a new row each time a number in the sequence ends with a 3 a pattern emerges:

3	7	11	15	19
23	27	31	35	39
43	47	51	55	59
63	67	71	75	79
83 ...				

As well as what is there, notice what is not there: there are no numbers that end in a 2 for example.

In fact, can you now see that there are no even numbers in the sequence ?

Good luck with the question...

Question 3

Let B = { The sequence of numbers that begins 3, 7, 11, 15, 19, 23, ... }
For each of the following, decide if the given statement is TRUE or FALSE.

(i) $15 \in B$

(ii) $9 \notin B$

(iii) $24 \in B$

(iv) $31 \notin B$

(v) $21 \in B$

(vi) $38 \in B$

(vii) $25 \in B$

(viii) $55 \in B$

(ix) $104 \in B$

(x) $526 \notin B$

[10 marks]

Question 4

Let,

A = { The first five triangular numbers }

B = { The factors of 15 }

C = { The ***prime*** factors of 390 }

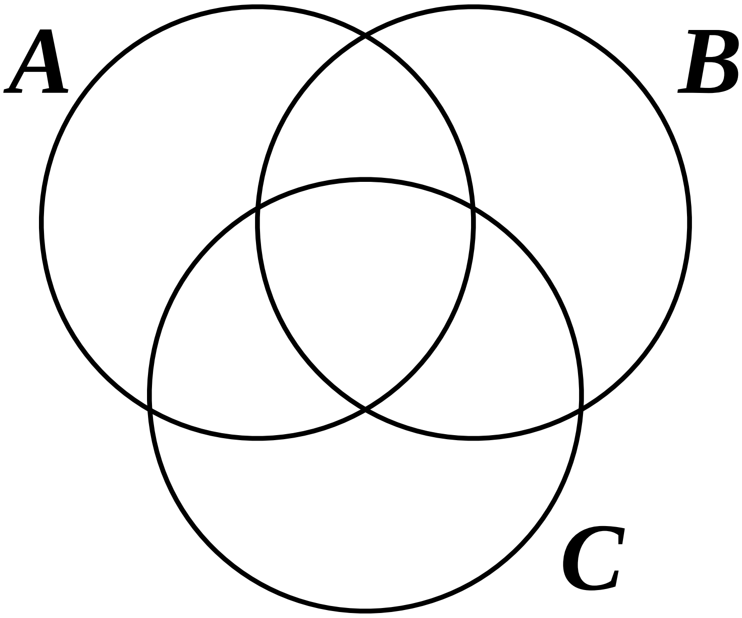
List the elements of sets *A*, *B* and *C* below,

A = { _____ , _____ , _____ , _____ , _____ }

B = { _____ , _____ , _____ , _____ }

C = { _____ , _____ , _____, _____ }

Use the three hoop Venn Diagram to show the relationship between sets *A*, *B* and *C*.



[10 marks]

Question 5

Let H ={ The sequence of numbers that begins 1, 5, 9, 13, ... }

Use the question 3 “Matched columns” technique to help answer the next question...
Begin by writing out the first 25 members of the set in the spaces below,

_____ , _____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ , _____ ,

For each of the following, decide if the given statement is TRUE or FALSE.

(i)	$13 \in H$	**(vi)**	$27 \in H$
(ii)	$19 \notin H$	**(vii)**	$63 \in H$
(iii)	$29 \in H$	**(viii)**	$4568 \in H$
(iv)	$18 \notin H$	**(ix)**	$91 \in H$
(v)	$85 \in H$	**(x)**	$207 \notin H$

[10 marks]

Question 6

Let,

D = { The factors of 36 }

E = { The first eight triangular numbers }

F = { The first six multiples of 6 }

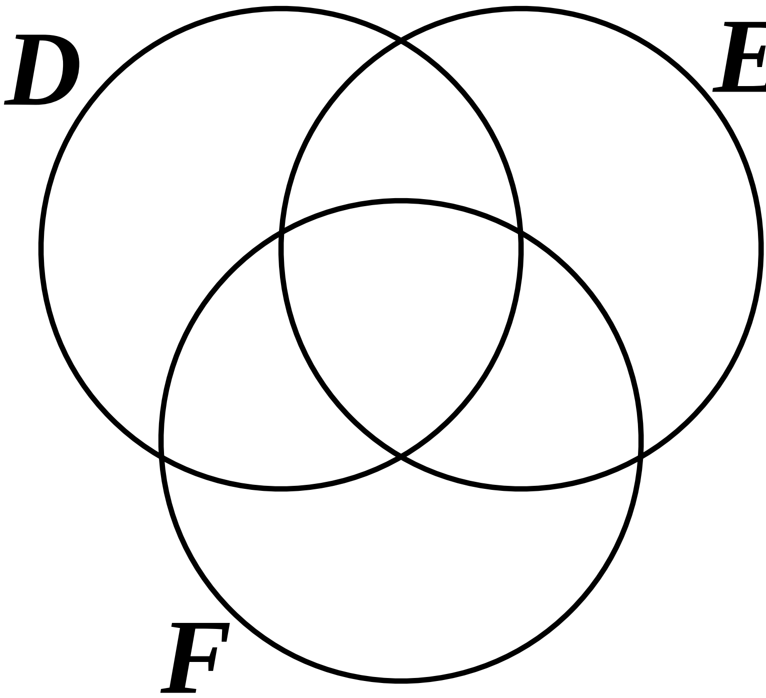
List the elements of sets *D*, *E* and *F* below,

D = { _____ , _____ , _____ , _____ , _____ , _____, _____ , _____ , _____}

E = { _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____ }

F = { _____ , _____ , _____, _____ , _____ , _____}

Use the Venn Diagram to show the relationship between sets D, E and F.



[10 marks]

Question 7

Let D ={ The sequence of numbers that begins 2, 4, 8, 16, ... }

Use the "Matched Columns" technique to help answer the next question.
Begin by writing out the first 16 members of the set in the spaces below,

__________ , __________ , __________ , __________ ,

__________ , __________ , __________ , __________ ,

__________ , __________ , __________ , __________ ,

__________ , __________ , __________ , __________ ,

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $42 \in D$

(ii) $32 \notin D$

(iii) $512 \in D$

(iv) $8008 \notin D$

(v) $2^3 \in D$

(vi) $2^5 \in D$

(vii) $2^{200} \in D$

(viii) $1\,200\,875 \in D$

(ix) $3^4 \in D$

(x) $2^{43} - 1 \notin D$

[10 marks]

Question 8

Let P ={ The sequence of numbers that begins 3, 6, 12, 24, ... }

Use the "Matched Columns" technique to help answer the next question...
Begin by writing out the first 13 members of the set in the spaces below,

__________ , __________ , __________ , __________ , __________ ,

__________ , __________ , __________ , __________ ,

__________ , __________ , __________ , __________ ,

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $3 \in P$

(ii) $33 \in P$

(iii) $33\,333 \in P$

(iv) $2 \times 3 \in P$

(v) $2^2 \times 3 \in P$

(vi) $2^3 \times 3 \in P$

(vii) $2^7 \times 3 \in P$

(viii) $12\,288 \in P$

(ix) $10^8 \in P$

(x) $2^{40} \notin P$

[10 marks]

Question 9
Apply the “Matched Columns” technique to the square numbers.
i.e. *S* = { 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ... }

You'll need to zig-zag the numbers.

________ , ________ , ________ , ________ , ________ ,

________ , ________ , ________ , ________ , ________ ,

________ , ________ , ________ , ________ , ________ ,

________ , ________ , ________ , ________ , ________ ,

________ , ________ , ________ , ________ , ________ ,

[4 marks]

What digits can a square number NOT end in ?

[4 marks]

Question 10
The “Matched Columns” trick does NOT work with triangular numbers. :-(

However, just as four endings are not possible with square numbers, four endings are not possible with triangular numbers.

Play around until you can see what those impossible endings are.

What digits can a triangular number NOT end in ?

[6 marks]

Question 11

2 is the least positive integer to have two factors, { 1, 2 }
4 is the least positive integer to have three factors, { 1, 2, 4 }
6 is the least positive integer to have four factors, { 1, 2, 3, 6 }

You might think that this *least integer to have n factors* sequence goes, 2, 4, 6, 8.
But don't jump to conclusions !
Show that 8 is not the least integer to have five factors.

What is the least integer to have five factors ?

The sequence is becoming even more weird...
What is the least integer to have six factors ?

Continue the sequence a little further, if you have time.

[6 marks]