Lesson 7

GCSE Mathematics
Set Theory I

7.1 Revision

Marks Available : 74

Question 1

Here are two descriptions of a set *F*;

- *F* = { The factors of 14 }
- *F* = { 1, 2, 7, 14 }

In a similar manner, give another description of the following sets

(i)   *S* = { Snapchat, Instagram, Tumblr, Twitter, FaceBook, ... }

[1 mark]

(ii)   *C* = { Playing pieces in the game of Chess }

[1 mark]

(iii)   *O* = { Odd Numbers }

[1 mark]

(iv)   *M* = { 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, ... }

[1 mark]

Question 2

Let Δ = { Triangular Numbers }

i.e. Δ = { 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, ... }

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $2 \in \Delta$ **(vi)** $136 \in \Delta$

(ii) $25 \notin \Delta$ **(vii)** $1 \notin \Delta$

(iii) $36 \in \Delta$ **(viii)** $40000.5 \in \Delta$

(iv) $91 \notin \Delta$ **(ix)** $\sqrt{100} \in \Delta$

(v) $120 \notin \Delta$ **(x)** $\pi \in \Delta$

[10 marks]

This next question is going to ask you whether certain numbers are in the sequence 2, 6, 10, 14, 18, 22, ... or not.

Each term is four more than the term before.

Here is a clever trick that lets you see what the numbers in this sequence can end in.

You write out quite a lot of the sequence but try to arrange it so that a pattern in the endings becomes obvious.

2	6	10	14	18
22	26	30	34	38
42	46	50	54	58
62	66	70	74	78
82 ...				

Now try the question:

Question 3

Let C = { The sequence of numbers that begins 2, 6, 10, 14, 18, 22, ... }

For each of the following, decide if the given statement is TRUE or FALSE.

(**i**) $14 \in C$ (**vi**) $387 \in C$

(**ii**) $8 \notin C$ (**vii**) $90 \in C$

(**iii**) $23 \in C$ (**viii**) $590 \in C$

(**iv**) $30 \notin C$ (**ix**) $102 \in C$

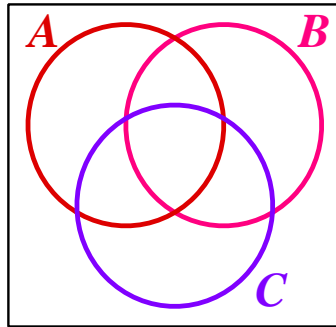(**v**) $20 \in C$ (**x**) $524 \notin C$
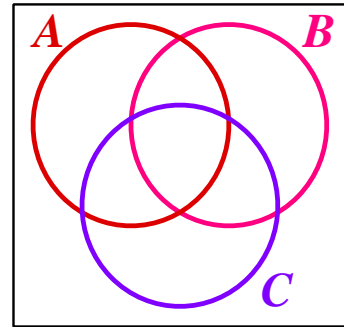
[10 marks]

Question 4
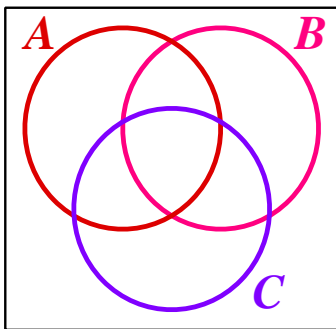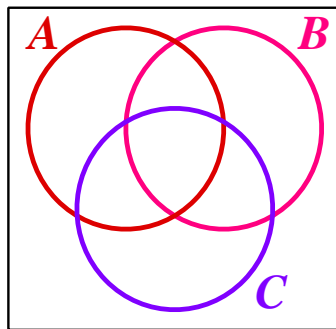
On the Venn Diagrams below, shade the region specified;
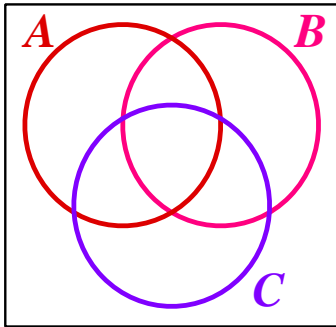
Shade : C

Shade : $C \cap B$
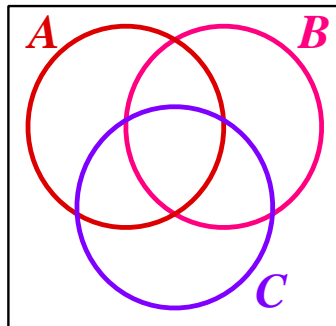
Shade : $C \cap B \cap A$

Shade :B'

Shade : $B' \cap C$
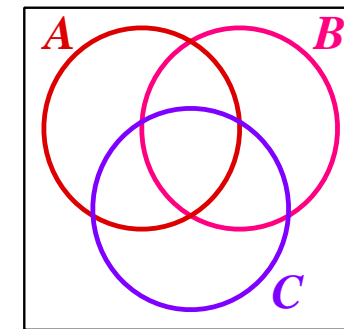
Shade : $B' \cap C \cap A$
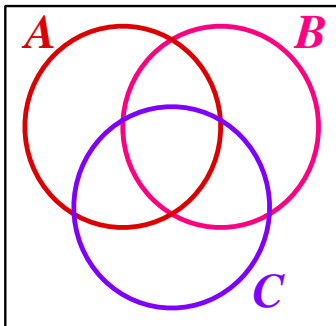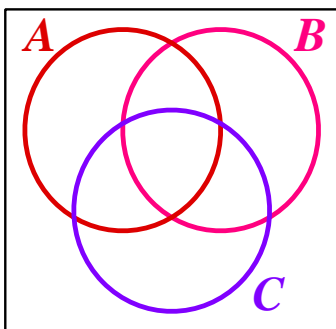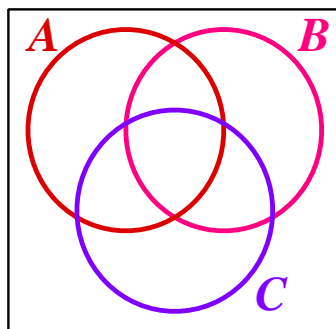
Shade : A

Shade : $A \cap C$
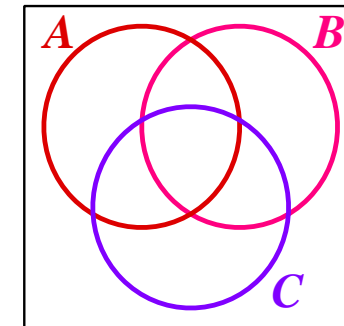
Shade : $(A \cap C)'$

Shade : $B' \cap C'$

Shade : $B' \cap C' \cap A$
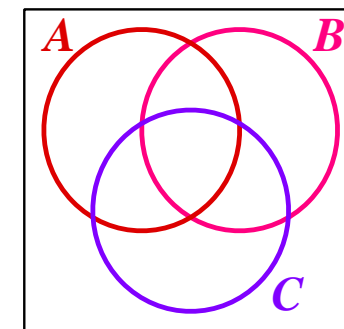
Shade : $C' \cap B' \cap A'$

[12 marks]

Question 5

Let *S*, *M* and *F* be the following sets;

      S = { The first five square numbers }
      M = { The first eight multiples of 4 }
      F = { The factors of 32 }

(a)   List the elements of sets S, M and F below,

     (i) S = { ____ , ____ , ____ , ____ , ____ }

                                                                           [1 mark]

     (ii) M = { ____ , ____ , ____ , ____ , ____ , ____ , ____ , ____ }

                                                                           [1 mark]

     (iii) F = { ____ , ____ , ____ , ____ , ____ , ____ }

                                                                           [1 mark]

(b)   Show the relationship between *S*, *M* and *F* on a Venn Diagram
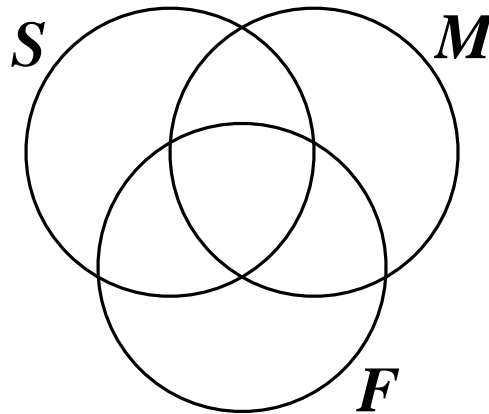
                                                                           [5 marks]

(c)   **(i)** $S \cap M$ = { ____ , ____ }

                                                                           [1 mark]

     (ii) $S \cap F$ = { ____ , ____ , ____ }

                                                                           [1 mark]

     (iii) $M \cap F$ = { ____ , ____ , ____ , ____ }

                                                                           [1 mark]

     (iv) $S \cap M \cap F$ = { ____ , ____ }

                                                                           [1 mark]

     (v) $S \cap F \cap M'$ = { ____ }

                                                                            [1 mark]

     (vi) $F \cap S' \cap M$ = { ____ , ____ }

                                                                           [1 mark]

     (vii)   What does the statement $S \cap M \cap F' = \emptyset$ tell you ?

                                                                            [1 mark]

Question 6

The Year 9 pupils in a school book club were asked if they enjoyed reading the Harry Potter books.
They were also asked if they enjoyed reading the His Dark Materials books.

The Venn Diagram provides a summary of their replies
where   • *P* is the hoop containing those who enjoyed the Harry Potter books
and   • *M* is the hoop containing those who enjoyed the His Dark Materials books
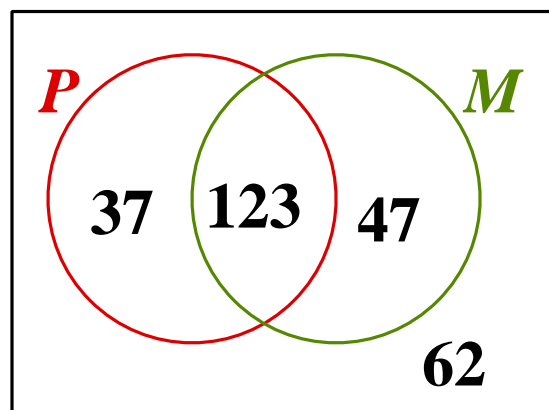
(**i**)   How many pupils were asked ?

[1 mark]

(**ii**)   How many pupils *only* enjoyed reading the Harry Potter books ?

[1 mark]

(**iii**)   How many pupils enjoyed reading the Harry Potter books ?

[1 mark]

(**iv**)   How many pupils didn't enjoy reading the Harry Potter books ?

[1 mark]

(**v**)   How many pupils enjoyed Harry Potter or His Dark Materials
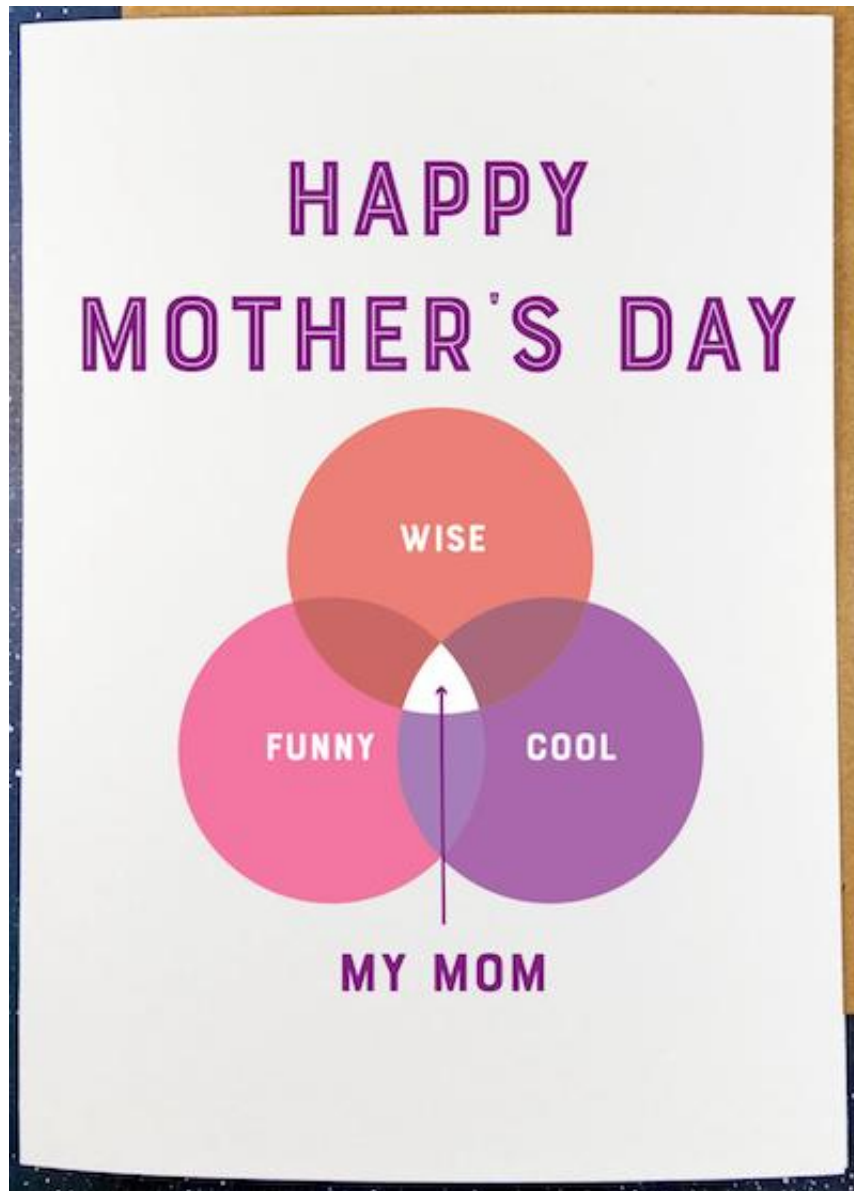   but not both ?

[1 mark]

Question 7

In this question, we are only working with the positive integers, {1, 2, 3, 4, 5, 6, 7, ... }

Let set O be the set of odd numbers; O = { 1, 3, 5, 7, 9, ... }

Describe, in words, the set O'

[2 marks]

Question 8

Explain what message this "Happy Mother's Day" card is conveying. ?

[2 marks]

Question 9

In this question, we are only working with the positive integers, {1, 2, 3, 4, 5, 6, 7, ... }

Let $S = \{$ The multiples of 7 $\}$

and $E = \{$ The multiples of 8 $\}$

(a)   List the first few elements of sets S and E below,

　　(i) $S = \{$ ____ , ____ , ____ , ____ , ____ , ____ , ____ , ____ , ... $\}$

　　(ii) $E = \{$ ____ , ____ , ____ , ____ , ____ , ____ , ____ , ____ , ...$\}$

[2 marks]

(b)   For each of the following, decide if the given statement is TRUE or FALSE

　　(i) $12 \notin E$ 　　　　**(vi)** $S \cap E = \{$ Multiples of 56 $\}$

　　(ii) $14 \in S$ 　　　　**(vii)** $S' = E'$

　　(iii) $56 \in S \cap E$ 　　　　**(viii)** $112 \in S' \cap E$

　　(iv) $560 \in S \cap E$ 　　　　**(ix)** $7^9 \in S$

　　(v) $21 \in (S \cap E)'$ 　　　　**(x)** $S' \cap E' = (S \cap E)'$

[10 marks]

(c)   **(i)**   On the first Venn diagrams below shade in $S' \cap E'$

　　(ii)   On the second Venn diagram below shade in $(S \cap E)'$

[2 marks]