$$x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 = x^0$$

THE

$$x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 = x^{11}$$

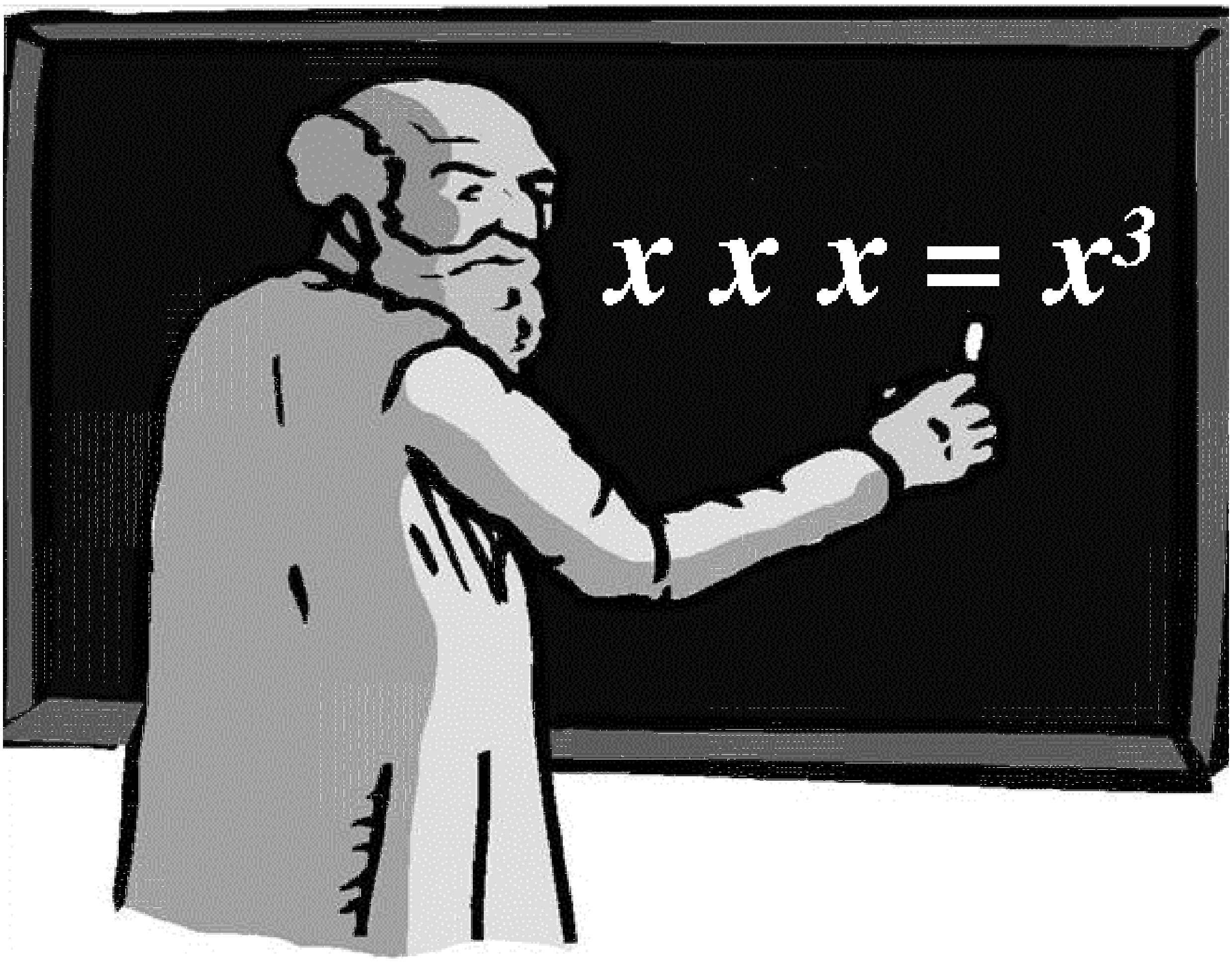
INDEX

$$x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 = x^{22}$$

FORM

$$x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 = x^{33}$$

Second Edition



Index Form

Lesson 1

GCSE (Year 9) Mathematics
Index Form

1.1 Numbers written in index form

Here is a number written in index form; 7^4

This is a quick way of representing,

$$7 \times 7 \times 7 \times 7$$

or, in other words,

$$2401$$

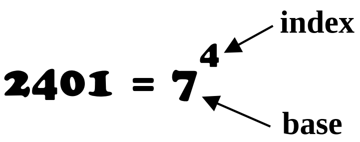
1.2 Base and Index

It's often convenient to write an integer, such as 2401, in index form,

$$2401 = 7^4$$

This is because the index form gives insight to the number 2401.
It makes it obvious, for example, that 2401 will divide exactly by 7 but not by 13.

When a number, such as 2401, is written as 7^4 mathematicians call the 7 the base of the number and the 4 the index.



1.3 Index Form Arithmetic

It's useful to be able to do arithmetic keeping the numbers involved in index form.

1.3.1 The 1st Law : When multiplying, same base indices add

$$\begin{aligned} & 7^3 \times 7^2 \\ = \; & \left(7^3\right) \times \left(7^2\right) \\ = \; & (7 \times 7 \times 7) \times (7 \times 7) \\ = \; & 7 \times 7 \times 7 \times 7 \times 7 \\ = \; & 7^5 \end{aligned}$$

$$\therefore 7^3 \times 7^2 = 7^5$$

The 1st Law

When multiplying, same base indices add,

$$a^m \times a^n = a^{m+n}$$

1.3.2 2nd Law : When dividing, same base indices subtract

$$\frac{7^5}{7^3}$$

$$= \frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7}$$

$$= \frac{(7 \times 7 \times 7) \times (7 \times 7)}{(7 \times 7 \times 7)}$$

$$= 7 \times 7$$

$$= 7^2$$

$$\therefore \frac{7^5}{7^3} = 7^2$$

The 2nd Law

When dividing, same base indices subtract,

$$\frac{a^m}{a^n} = a^{m-n}$$

1.3.3 3rd Law : When powering a power, indices multiply

power

$$2401 = 7^4$$

An index can also be called a power

$$(7^3)^2$$

$$= (7^3) \times (7^3)$$

$$= (7 \times 7 \times 7) \times (7 \times 7 \times 7)$$

$$= 7 \times 7 \times 7 \times 7 \times 7 \times 7$$

$$= 7^6$$

$$\therefore (7^3)^2 = 7^6$$

The 3rd Law

When powering an power, indices multiply

$$(a^m)^n = a^{mn}$$

1.3.4 4th Law : A square root halves the index

$$\sqrt{7^6}$$

$$= \sqrt{7 \times 7 \times 7 \times 7 \times 7 \times 7}$$

$$= \sqrt{(7 \times 7 \times 7) \times (7 \times 7 \times 7)}$$

$$= (7 \times 7 \times 7)$$

$$= 7^3$$

$$\therefore \sqrt{7^6} = 7^3$$

The 4th Law

A square root halves the index,

$$\sqrt{a^m} = a^{\frac{m}{2}}$$

With $m = 1$, the 4th law tells us that,

A 4th Law Consequence

A square root can be replaced with an index of $\frac{1}{2}$

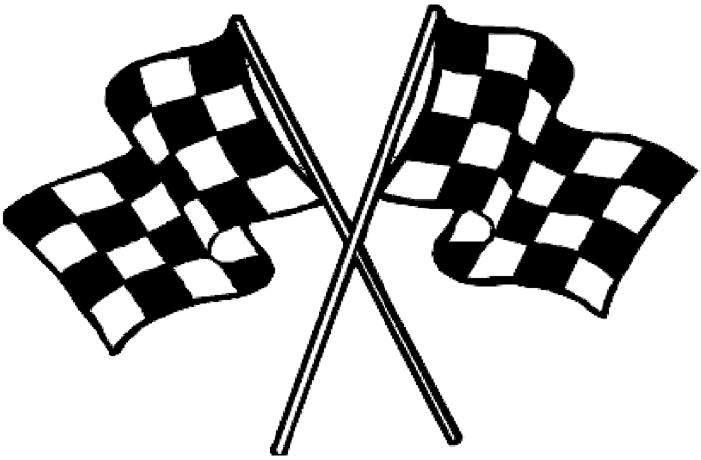
$$\sqrt{a} = a^{\frac{1}{2}}$$

1.4 **Exercise**

GCSE (Year 9) Mathematics
Index Form

Index Form Race N° 1

Do NOT use a calculator



Write answers in prime index form, p^m, for some prime, p, and some integer, m

Target time : 15 minutes

(a) $13^5 \times 13^4$

(b) $7^8 \times 7^6$

(c) 11×11^4

(d) $5^4 \times 5^4 \times 5^4$

(e) $\left(5^7\right)^3$

(f) $\left(2^7\right)^5$

(g) $101^{20} \times 101^5$

(h) $5^{82} \times 5^{54}$

(i) $\left(3^4\right)^2 \times \left(3^2\right)^5$

(j) $\dfrac{7^8}{7^3}$

(k) $\dfrac{5^{50}}{5^{25}}$

(l) $\dfrac{11^7}{11}$

(m) $\dfrac{13^5}{13^5}$

(n) $\dfrac{103^{50}}{103^{10}}$

(o) $\dfrac{7^{102}}{7^{51}}$

(p) $\left(3^{2}\right)^{3} \times 3^{5}$

(q) $\left(5^{4}\right)^{3} \times 5^{2}$

(r) $\left(23^{7}\right)^{2} \times 23^{5}$

(s) $\dfrac{2^{7}}{2^{4}} \times \dfrac{2^{5}}{2^{3}}$

(t) $\dfrac{5^{6}}{5^{4}} \times \dfrac{5^{7}}{5^{2}}$

(u) $\sqrt{5^{8}}$

(v) $\dfrac{7^{11}}{7^{4}} \times \dfrac{7^{6}}{7^{3}}$

(w) $\dfrac{3^{5} \times 3^{4}}{3^{2} \times 3^{3}}$

(x) $\dfrac{\left(5^{3}\right)^{4}}{\left(5^{2}\right)^{3}}$

(y) $\sqrt{11^{6}}$

(z) $\sqrt{\dfrac{5^{11}}{5^{5}}}$