

$$x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 \times x^0 = x^0$$

T H E

$$x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 \times x^1 = x^{11}$$

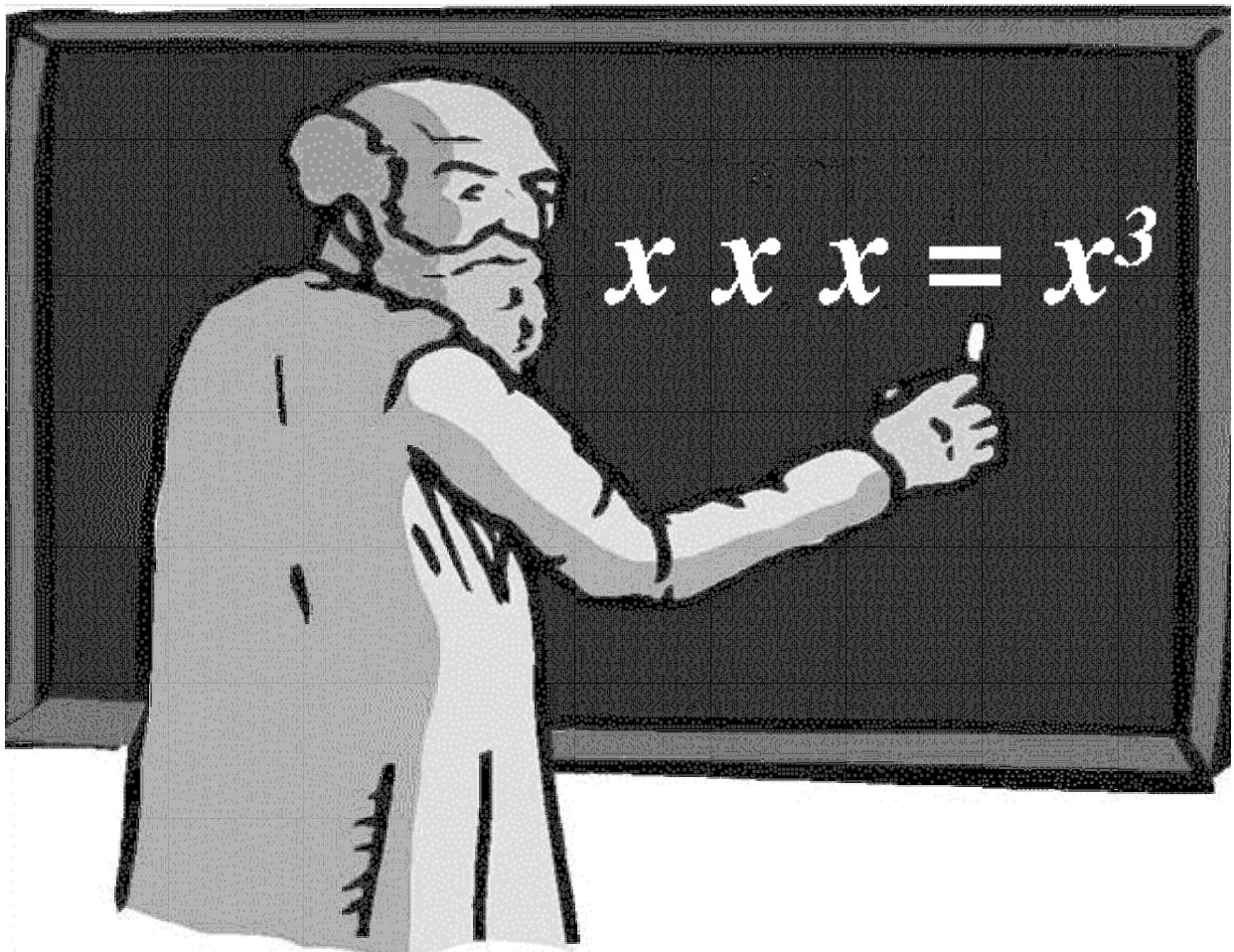
I N D E X

$$x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 \times x^2 = x^{22}$$

F O R M

$$x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 \times x^3 = x^{33}$$

Second Edition



Index Form

Lesson 1

GCSE (Year 9) Mathematics Index Form

1.1 Numbers written in index form

Here is a number written in index form; 7^4

This is a quick way of representing,

$$7 \times 7 \times 7 \times 7$$

or, in other words,

$$2401$$

1.2 Base and Index

It's often convenient to write an integer, such as 2401, in index form,

$$2401 = 7^4$$

This is because the index form gives insight to the number 2401.

It makes it obvious, for example, that 2401 will divide exactly by 7 but not by 13.

When a number, such as 2401, is written as 7^4 mathematicians call the 7 the base of the number and the 4 the index.

$$\mathbf{2401 = 7^4}$$

index
↙
↘
base

1.3 Index Form Arithmetic

It's useful to be able to do arithmetic keeping the numbers involved in index form.

1.3.1 The 1st Law : When multiplying, same base indices add

$$\begin{aligned} & 7^3 \times 7^2 \\ &= (7^3) \times (7^2) \\ &= (7 \times 7 \times 7) \times (7 \times 7) \\ &= 7 \times 7 \times 7 \times 7 \times 7 \\ &= 7^5 \\ &\therefore 7^3 \times 7^2 = 7^5 \end{aligned}$$

The 1st Law

When multiplying, same base indices add,

$$a^m \times a^n = a^{m+n}$$

1.3.2 2nd Law : When dividing, same base indices subtract

$$\begin{aligned} & \frac{7^5}{7^3} \\ &= \frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7} \\ &= \frac{(7 \times 7 \times 7) \times (7 \times 7)}{(7 \times 7 \times 7)} \\ &= 7 \times 7 \\ &= 7^2 \\ &\therefore \frac{7^5}{7^3} = 7^2 \end{aligned}$$

The 2nd Law

When dividing, same base indices subtract,

$$\frac{a^m}{a^n} = a^{m-n}$$

1.3.3 3rd Law : When powering a power, indices multiply

$$\mathbf{2401 = 7^4}$$

power ↙

An index can also be called a power

$$\begin{aligned} & (7^3)^2 \\ &= (7^3) \times (7^3) \\ &= (7 \times 7 \times 7) \times (7 \times 7 \times 7) \\ &= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \\ &= 7^6 \\ &\therefore (7^3)^2 = 7^6 \end{aligned}$$

The 3rd Law

When powering an power, indices multiply

$$(a^m)^n = a^{mn}$$

1.3.4 4th Law : A square root halves the index

$$\begin{aligned}\sqrt{7^6} &= \sqrt{7 \times 7 \times 7 \times 7 \times 7 \times 7} \\ &= \sqrt{(7 \times 7 \times 7) \times (7 \times 7 \times 7)} \\ &= (7 \times 7 \times 7) \\ &= 7^3 \\ \therefore \sqrt{7^6} &= 7^3\end{aligned}$$

The 4th Law

A square root halves the index,

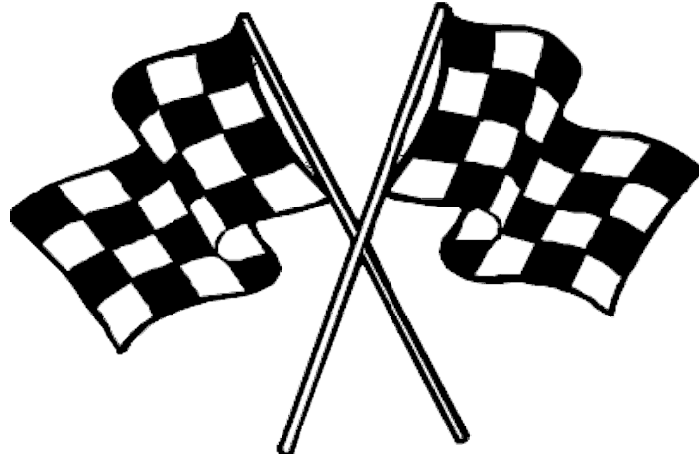
$$\sqrt{a^m} = a^{\frac{m}{2}}$$

With $m = 1$, the 4th law tells us that,

A 4th Law Consequence

A square root can be replaced with an index of $\frac{1}{2}$

$$\sqrt{a} = a^{\frac{1}{2}}$$

Index Form Race N° 1*Do NOT use a calculator*Write answers in prime index form, p^m , for some prime, p , and some integer, m *Target time : 15 minutes*

(a) $13^5 \times 13^4$

(b) $7^8 \times 7^6$

(c) 11×11^4

(d) $5^4 \times 5^4 \times 5^4$

(e) $(5^7)^3$

(f) $(2^7)^5$

(g) $101^{20} \times 101^5$

(h) $5^{82} \times 5^{54}$

(i) $(3^4)^2 \times (3^2)^5$

(j) $\frac{7^8}{7^3}$

(k) $\frac{5^{50}}{5^{25}}$

(l) $\frac{11^7}{11}$

(m) $\frac{13^5}{13^5}$

(n) $\frac{103^{50}}{103^{10}}$

(o) $\frac{7^{102}}{7^{51}}$

$$(p) \quad (3^2)^3 \times 3^5$$

$$(q) \quad (5^4)^3 \times 5^2$$

$$(r) \quad (23^7)^2 \times 23^5$$

$$(s) \quad \frac{2^7}{2^4} \times \frac{2^5}{2^3}$$

$$(t) \quad \frac{5^6}{5^4} \times \frac{5^7}{5^2}$$

$$(u) \quad \sqrt{5^8}$$

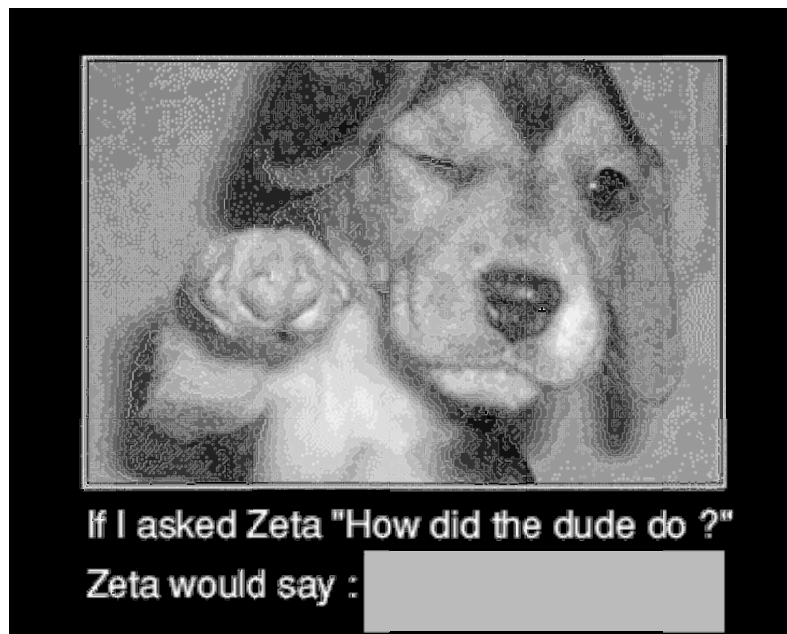
$$(v) \quad \frac{7^{11}}{7^4} \times \frac{7^6}{7^3}$$

$$(w) \quad \frac{3^5 \times 3^4}{3^2 \times 3^3}$$

$$(x) \quad \frac{(5^3)^4}{(5^2)^3}$$

$$(y) \quad \sqrt{11^6}$$

$$(z) \quad \sqrt{\frac{5^{11}}{5^5}}$$



This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

© 2021 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk