2.1 The First Four Laws

In lesson 1, four Laws concerning numbers written in index form were deduced,

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The 1st Law

When multiplying, same base indices add,

$$a^m \times a^n = a^{m+n}$$

The 2nd Law

When dividing, same base indices subtract,

$$\frac{a^m}{a^n} = a^{m-n}$$

The 3rd Law

When powering a power, indices multiply,

$$\left(a^m\right)^n = a^{mn}$$

The 4th Law

A square root halves the index,

$$\sqrt{a^m} = a^{\frac{m}{2}}$$

2.2 An Index of Half

An interesting consequence of the 4th Law was deduced by letting $m = 1$.
This gave a result which is sufficiently important to make it the 5th Law

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5th Law

A square root can be replaced with an index of $\frac{1}{2}$

$$\sqrt{a} = a^{\frac{1}{2}}$$

The following clever calculation, in which the number 16 is rewritten as 2^4, shows that the 5th law can be deduced from the 3rd,

$$\begin{aligned} 16^{\frac{1}{2}} &= \left(2^4\right)^{\frac{1}{2}} \\ &= 2^2 \\ &= 4 \end{aligned}$$

2.3 An Index of Zero

The 2nd Law also gives rise to a surprising result.
Recall that it says that "When dividing, same base indices subtract"
For example,

$$\frac{7^5}{7^3} = 7^2$$

Now consider the similar calculation $\frac{7^4}{7^4}$ in two different ways:

- ◇ Firstly, pretty much anything divided by itself is 1,

$$\frac{7^4}{7^4} = 1$$

- ◇ Secondly, by the 2nd Law,

$$\begin{aligned} \frac{7^4}{7^4} &= 7^{4-4} \\ &= 7^0 \end{aligned}$$

It is said that "Mathematics abhors a contradiction" by which is meant that the same question tackled in different ways should lead to equivalent answers.
So, the two seemingly different answers must in fact equal each other.

That is, $7^0 = 1$

6th Law

Any real number to the power zero equals one

(with the sole exception of 0^0 which is undefined)

$$a^0 = 1 \qquad a \neq 0$$

Example

$$\begin{aligned} & 11^5 \times 11^0 \\ = \; & 11^5 \times 1 \qquad \text{using the 6th Law} \\ = \; & 11^5 \end{aligned}$$

This example could have also been done using the 1st Law.

2.4 Exercise

Index Form Race N° 2

Do NOT use a calculator

Write answers in prime index form, p^m, for some prime, p, and some real number, m

Target time : 15 minutes

(a) $17^{15} \times 17^{10}$ **(b)** $7^8 \times 7^0$ **(c)** $107^0 \times 107^{\frac{1}{2}}$

(d) $5^4 \times 5$ **(e)** $\left(5^4\right)^6$ **(f)** $\left(5^4\right)^{\frac{1}{2}}$

(g) $\left(13^{\frac{1}{2}}\right)^{12}$ **(h)** $7^{\frac{1}{2}} \times 7^{\frac{1}{2}}$ **(i)** $\left(17^4\right)^{\frac{1}{2}} \times \left(17^2\right)^4$

(j) $\frac{11^{13}}{11^8}$ **(k)** $\frac{\sqrt{7}}{7^{\frac{1}{2}}}$ **(l)** $\sqrt{5} \times \sqrt{5}$

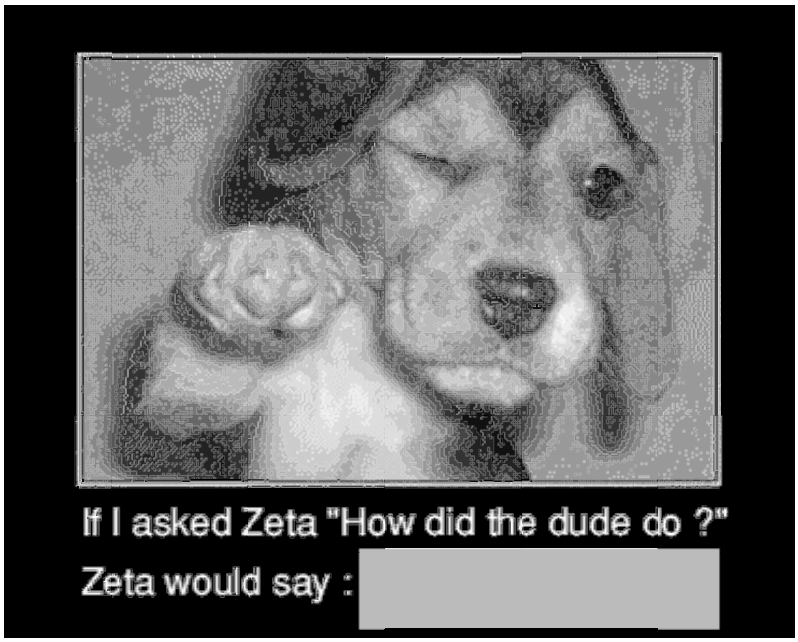
(m) $\frac{17^{\frac{1}{2}}}{17^{\frac{1}{2}}}$ **(n)** $\frac{23^{0.5}}{23^0}$ **(o)** $19^0 \times 19$

(p) $\left(29^{20}\right)^3 \times 29^7$ **(q)** $\left(7^{10}\right)^{\frac{1}{2}} \times \frac{7^{14}}{7^3}$ **(r)** $\left(2^8\right)^8 \times 2^5$

(s) $\dfrac{5^7}{5^2} \times \dfrac{5^0}{5^3}$ **(t)** $\dfrac{13^{14} \times 13^6}{13^2 \times 13^8}$ **(u)** $\sqrt{41^{22}}$

(v) $\dfrac{7^1}{7^{\frac{1}{2}}} \times \dfrac{7^2}{7^{\frac{1}{2}}}$ **(w)** $\left(\sqrt{13^6}\right)^0$ **(x)** $\dfrac{\left(5^8\right)^4}{\left(5^2\right)^5}$

(y) $\sqrt{19^{56}}$ **(z)** $\sqrt{\dfrac{\left(5^4\right)^{11}}{5^6}}$



2.5 Exercise

Index Form Race N° 3

Do NOT use a calculator

Write answers in prime index form, p^m, for some prime, p, and some real number, m

Target time : 15 minutes

(a) $17^{50} \times 17^{60}$ **(b)** $\sqrt{13^{48}}$ **(c)** $11^{45} \times 11^{0}$

(d) $\dfrac{19^{33}}{19^{11}}$ **(e)** $23^{25} \times 23^{4}$ **(f)** $\left(13^{44}\right)^{2}$

(g) $\left(\sqrt{3}\right)^{8}$ **(h)** $p^{21} \times p^{53}$ **(i)** $\sqrt{\sqrt{5^{20}}}$

(j) $\dfrac{p^{36}}{p^{4}}$ **(k)** $\left(p^{5}\right)^{10}$ **(l)** $\dfrac{47^{7}}{47}$

(m) $p^{53} \times p$ **(n)** $\dfrac{\left(7^{8}\right)^{5}}{\left(7^{6}\right)^{3}}$ **(o)** $\dfrac{p^{25}}{p}$

(p) $\left(p^{9}\right)^{3} \times p^{23}$ **(q)** $\left(17^{20}\right)^{\frac{1}{2}} \times \dfrac{17^{12}}{17^{3}}$ **(*r*)** $\left(31^{7}\right)^{7} \times 31^{5}$

(s) $\dfrac{p^9}{p^2} \times \dfrac{p^7}{p^6}$ **(t)** $\dfrac{p^5 \times p^7}{p^8}$ **(u)** $\sqrt{p^{24}}$

(v) $\dfrac{7^{10}}{7^{\frac{1}{2}}} \times \dfrac{7^{20}}{7^{\frac{1}{2}}}$ **(w)** $p^{\frac{1}{2}} \times p^{\frac{1}{2}}$ **(x)** $\dfrac{(p^7)^4}{(p^2)^6}$

(y) $\sqrt{17^{12} \times 17^{26}}$ **(z)** $\sqrt{\dfrac{(23^2)^{14}}{23^8}}$