3.1 Negative Indices

In lesson 2, a calculation done in two different ways resulted in seemingly different answers. Mathematical logic then dictated that each had be equal to the other. The result was the 6th Law of indices,

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6th Law

Any real number to the power zero equals one

(with the sole exception of 0^0 which is undefined)

$$a^0 = 1 \qquad a \neq 0$$

This "two different paths" technique is frequently employed by mathematicians. What follows is another example of its use.

Consider the following chain of reasoning,

$$\frac{7^3}{7^5}$$

$$= \frac{7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7}$$

$$= \frac{(7 \times 7 \times 7)}{(7 \times 7 \times 7) \times (7 \times 7)}$$

$$= \frac{1}{7 \times 7}$$

$$= \frac{1}{7^2}$$

Now, look at this alternative processing of the same calculation,

$$\frac{7^3}{7^5} = 7^{3-5} \qquad (\text{By the } 2^{\text{nd}} \textit{ Law})$$

$$= 7^{-2}$$

The inescapable conclusion is that,

$$7^{-2} = \frac{1}{7^2}$$

7th Law

A negative index means reciprocal

$$a^{-m} = \frac{1}{a^m} \qquad a \neq 0$$

3.2 Exercise

Index Form Race N° 5

Do NOT use a calculator

Write answers in prime index form, p^m, for some prime, p, and some real number, m

Target time : 15 minutes

(a) $\frac{1}{5^8}$ **(b)** $\frac{1}{3^7}$ **(c)** $\frac{1}{7}$

(d) $5^9 \times 5^{-4}$ **(e)** $13^{13} \times 13^{-3}$ **(f)** $7^5 \times 7^{-13}$

(g) $2^9 \times 2^{-9}$ **(h)** $\frac{1}{11^5}$ **(i)** 2×2^{-7}

(j) $\frac{11^8}{11^5}$ **(k)** $\frac{7^5}{7^{11}}$ **(l)** $\frac{17^7}{17^{13}}$

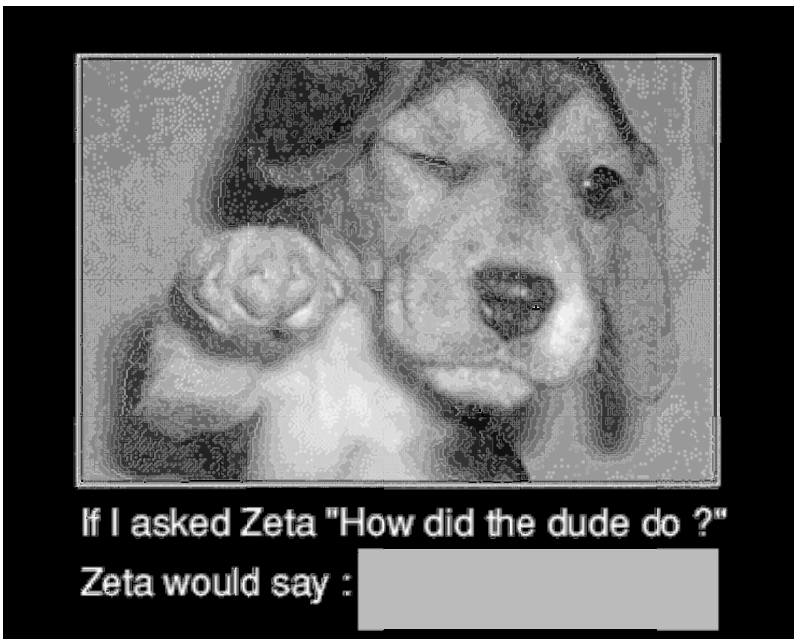
(m) $\frac{13^{-3}}{13^6}$ **(n)** $\sqrt{7^{-12}}$ **(o)** $\left(5^{-8}\right)^2$

(p) $\left(11^2\right)^{-3}$ **(q)** $\left(7^{-10}\right)^{-5}$ **(r)** $\left(2^7\right)^7 \times 2^{-25}$

(s) $\frac{5^7}{5^3} \times \frac{5^{-2}}{5^0}$ **(t)** $\frac{1}{(3^4)^{\frac{1}{2}}}$ **(u)** $\sqrt{5^{-2}}$

(v) $\frac{1}{2^3}$ **(w)** $\frac{1}{2^{-2}}$ **(x)** $\frac{\left(5^2\right)^4}{\left(5^5\right)^3}$

(y) $\sqrt{17^{-26}}$ **(z)** $\sqrt{\frac{(2^3)^{11}}{2^{55}}}$



3.3 Exercise

Index Form Race N° 6

Do NOT use a calculator

Write answers in prime index form, p^m, for some prime, p, and some real number, m

Target time : 15 minutes

(a) $\frac{1}{7^9}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{7^{-4}}$

(d) $7^{17} \times 7^{-14}$ **(e)** $11^{-4} \times 11^{-6}$ **(f)** $13^{15} \times 13^{-25}$

(g) $19^{13} \times 19^{-12}$ **(h)** $\frac{1}{17^5}$ **(i)** 7×7^{-17}

(j) $\frac{13^{15}}{13^{18}}$ **(k)** $\frac{11^5}{11^{34}}$ **(l)** $\frac{7^{100}}{7^{101}}$

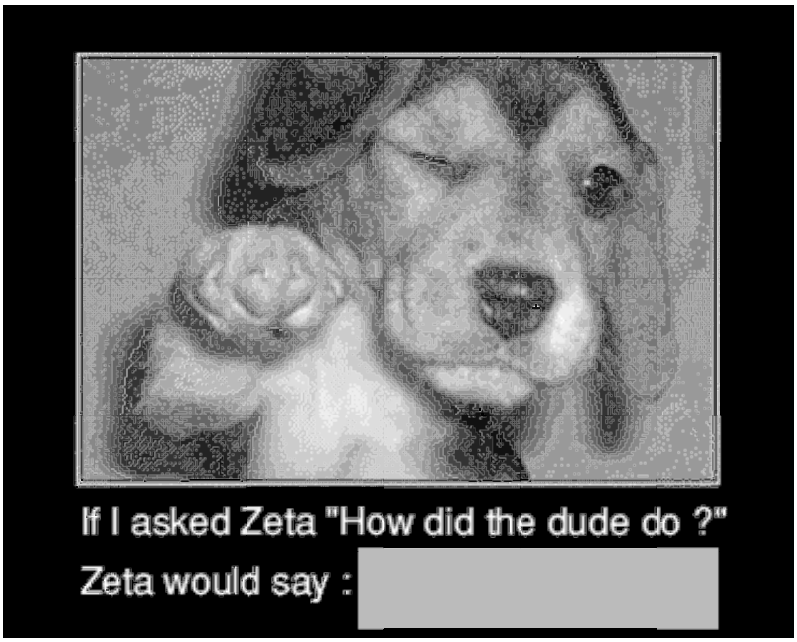
(m) $\frac{17^{-8}}{17^5}$ **(n)** $\sqrt{17^{-24}}$ **(o)** $\left(7^{-6}\right)^5$

(p) $\left(31^{22}\right)^{-4}$ **(q)** $\left(17^{-12}\right)^{-5}$ **(r)** $\left(3^{5}\right)^{5} \times 3^{-25}$

(s) 4×2^{-5} **(t)** $\dfrac{3^{-5}}{\left(3^{4}\right)^{\frac{1}{2}}}$ **(u)** $\sqrt{\left(7^{-3}\right)^{-6}}$

(v) $\dfrac{1}{23^{4}}$ **(w)** $\dfrac{1}{23^{-4}}$ **(x)** $\dfrac{\left(7^{3}\right)^{5}}{\left(7^{6}\right)^{6}}$

(y) $\sqrt{47^{-206}}$ **(z)** $\sqrt{\dfrac{\left(2^{-3}\right)^{11}}{2^{55}}}$



3.4 Exercise

Index Form Race N° 7

Do NOT use a calculator

Write answers in prime index form, p^m, for some prime, p, and some real number, m

Target time : 15 minutes

(a) $\frac{1}{p^{12}}$ **(b)** $\frac{1}{p}$ **(c)** $\frac{1}{p^{-7}}$

(d) $p^7 \times p^{-4}$ **(e)** $p^{-3} \times p^{-5}$ **(f)** $p^8 \times p^{-13}$

(g) $p^{-5} \times p^2$ **(h)** $\frac{1}{p^7}$ **(i)** $p \times p^{-1}$

(j) $\frac{p^{12}}{p^{19}}$ **(k)** $\frac{p^8}{p^{14}}$ **(l)** $\frac{p^{20}}{p^{31}}$

(m) $\frac{p^{-7}}{p^6}$ **(n)** $\sqrt{p^{-4}}$ **(o)** $\left(p^{-3}\right)^8$

(p) $\left(p^{33}\right)^{-3}$ **(q)** $\left(p^{-13}\right)^{-3}$ **(r)** $\left(p^{7}\right)^{3} \times p^{-25}$

(s) $p^{-\frac{1}{2}} \times p^{-\frac{1}{2}}$ **(t)** $\dfrac{p^{-4}}{\left(p^{6}\right)^{\frac{1}{2}}}$ **(u)** $\sqrt{\left(p^{-5}\right)^{-8}}$

(v) $\dfrac{1}{p^{5}}$ **(*w*)** $\dfrac{1}{p^{-5}}$ **(x)** $\dfrac{\left(p^{5}\right)^{5}}{\left(p^{6}\right)^{6}}$

(y) $\sqrt{p^{-888}}$ **(z)** $\sqrt{\dfrac{\left(p^{-3}\right)^{-11}}{p^{-55}}}$