

**3.1 Negative Indices**

In lesson 2, a calculation done in two different ways resulted in seemingly different answers. Mathematical logic then dictated that each had be equal to the other. The result was the 6<sup>th</sup> Law of indices,

**6<sup>th</sup> Law**

Any real number to the power zero equals one

(with the sole exception of  $0^0$  which is undefined)

$$a^0 = 1 \quad a \neq 0$$

This “two different paths” technique is frequently employed by mathematicians. What follows is another example of its use.

Consider the following chain of reasoning,

$$\begin{aligned} & \frac{7^3}{7^5} \\ &= \frac{7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7} \\ &= \frac{(7 \times 7 \times 7)}{(7 \times 7 \times 7) \times (7 \times 7)} \\ &= \frac{1}{7 \times 7} \\ &= \frac{1}{7^2} \end{aligned}$$

Now, look at this alternative processing of the same calculation,

$$\begin{aligned} \frac{7^3}{7^5} &= 7^{3-5} \quad (\text{By the 2<sup>nd</sup> Law}) \\ &= 7^{-2} \end{aligned}$$

The inescapable conclusion is that,

$$7^{-2} = \frac{1}{7^2}$$

**7<sup>th</sup> Law**

A negative index means reciprocal

$$a^{-m} = \frac{1}{a^m} \quad a \neq 0$$

### 3.2 Exercise

## Index Form Race N° 5

Do NOT use a calculator



Write answers in prime index form,  $p^m$ , for some prime,  $p$ , and some real number,  $m$

*Target time : 15 minutes*

(a)  $\frac{1}{5^8}$

(b)  $\frac{1}{3^7}$

(c)  $\frac{1}{7}$

(d)  $5^9 \times 5^{-4}$

(e)  $13^{13} \times 13^{-3}$

(f)  $7^5 \times 7^{-13}$

(g)  $2^9 \times 2^{-9}$

(h)  $\frac{1}{11^5}$

(i)  $2 \times 2^{-7}$

(j)  $\frac{11^8}{11^5}$

(k)  $\frac{7^5}{7^{11}}$

(l)  $\frac{17^7}{17^{13}}$

(m)  $\frac{13^{-3}}{13^6}$

(n)  $\sqrt{7^{-12}}$

(o)  $(5^{-8})^2$

$$(p) \quad (11^2)^{-3}$$

$$(q) \quad (7^{-10})^{-5}$$

$$(r) \quad (2^7)^7 \times 2^{-25}$$

$$(s) \quad \frac{5^7}{5^3} \times \frac{5^{-2}}{5^0}$$

$$(t) \quad \frac{1}{(3^4)^{\frac{1}{2}}}$$

$$(u) \quad \sqrt{5^{-2}}$$

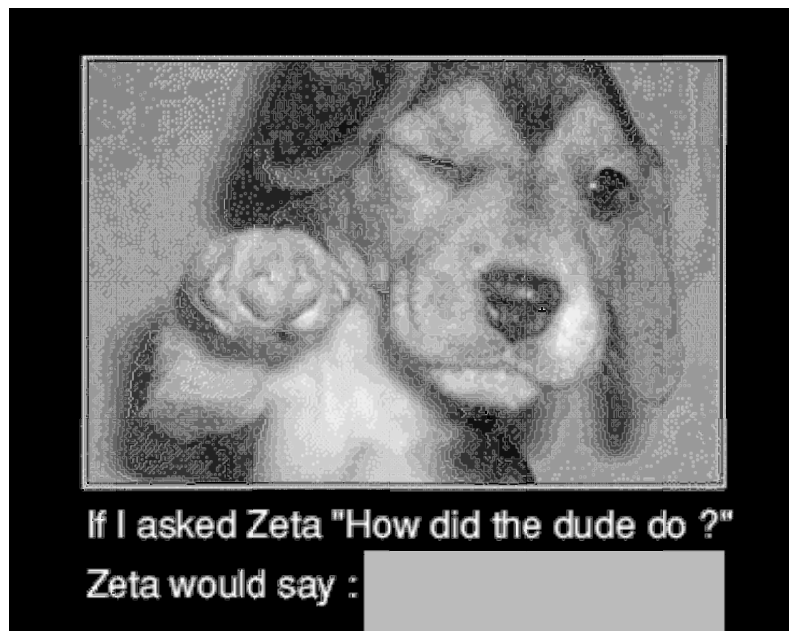
$$(v) \quad \frac{1}{2^3}$$

$$(w) \quad \frac{1}{2^{-2}}$$

$$(x) \quad \frac{(5^2)^4}{(5^5)^3}$$

$$(y) \quad \sqrt{17^{-26}}$$

$$(z) \quad \sqrt{\frac{(2^3)^{11}}{2^{55}}}$$



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### 3.3 Exercise

## Index Form Race N° 6

*Do NOT use a calculator*



Write answers in prime index form,  $p^m$ , for some prime,  $p$ , and some real number,  $m$

*Target time : 15 minutes*

(a)  $\frac{1}{7^9}$

(b)  $\frac{1}{3}$

(c)  $\frac{1}{7^{-4}}$

(d)  $7^{17} \times 7^{-14}$

(e)  $11^{-4} \times 11^{-6}$

(f)  $13^{15} \times 13^{-25}$

(g)  $19^{13} \times 19^{-12}$

(h)  $\frac{1}{17^5}$

(i)  $7 \times 7^{-17}$

(j)  $\frac{13^{15}}{13^{18}}$

(k)  $\frac{11^5}{11^{34}}$

(l)  $\frac{7^{100}}{7^{101}}$

(m)  $\frac{17^{-8}}{17^5}$

(n)  $\sqrt{17^{-24}}$

(o)  $(7^{-6})^5$

(p)  $(31^{22})^{-4}$

(q)  $(17^{-12})^{-5}$

(r)  $(3^5)^5 \times 3^{-25}$

(s)  $4 \times 2^{-5}$

(t)  $\frac{3^{-5}}{(3^4)^{\frac{1}{2}}}$

(u)  $\sqrt{(7^{-3})^{-6}}$

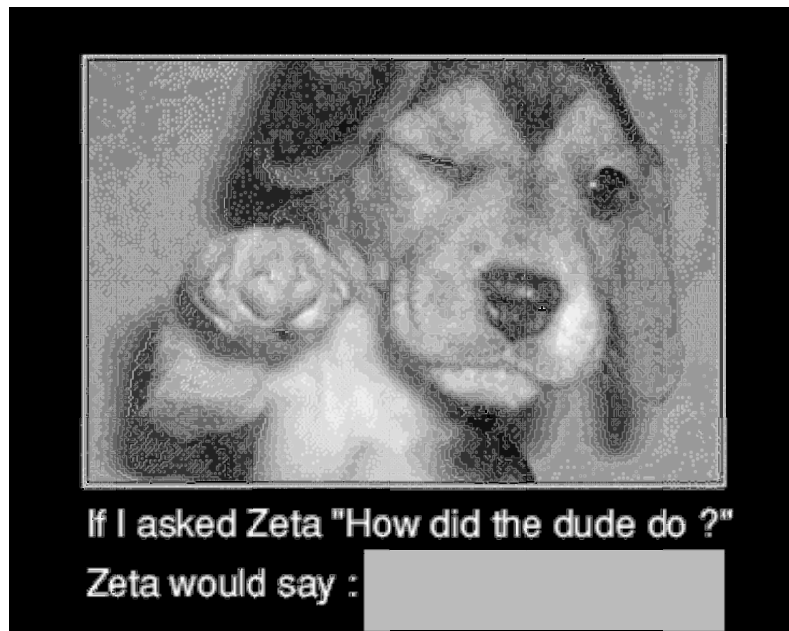
(v)  $\frac{1}{23^4}$

(w)  $\frac{1}{23^{-4}}$

(x)  $\frac{(7^3)^5}{(7^6)^6}$

(y)  $\sqrt{47^{-206}}$

(z)  $\sqrt{\frac{(2^{-3})^{11}}{2^{55}}}$



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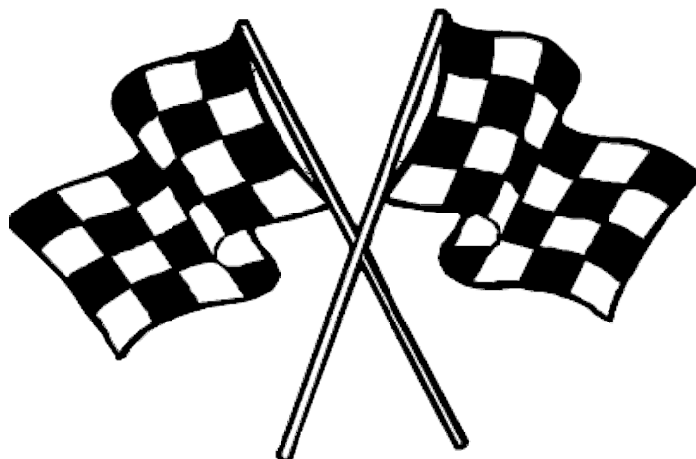
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### 3.4 Exercise

## Index Form Race N° 7

*Do NOT use a calculator*



Write answers in prime index form,  $p^m$ , for some prime,  $p$ , and some real number,  $m$

*Target time : 15 minutes*

(a)  $\frac{1}{p^{12}}$

(b)  $\frac{1}{p}$

(c)  $\frac{1}{p^{-7}}$

(d)  $p^7 \times p^{-4}$

(e)  $p^{-3} \times p^{-5}$

(f)  $p^8 \times p^{-13}$

(g)  $p^{-5} \times p^2$

(h)  $\frac{1}{p^7}$

(i)  $p \times p^{-1}$

(j)  $\frac{p^{12}}{p^{19}}$

(k)  $\frac{p^8}{p^{14}}$

(l)  $\frac{p^{20}}{p^{31}}$

(m)  $\frac{p^{-7}}{p^6}$

(n)  $\sqrt{p^{-4}}$

(o)  $(p^{-3})^8$

$$(p) \quad (p^{33})^{-3}$$

$$(q) \quad (p^{-13})^{-3}$$

$$(r) \quad (p^7)^3 \times p^{-25}$$

$$(s) \quad p^{-\frac{1}{2}} \times p^{-\frac{1}{2}}$$

$$(t) \quad \frac{p^{-4}}{(p^6)^{\frac{1}{2}}}$$

$$(u) \quad \sqrt{(p^{-5})^{-8}}$$

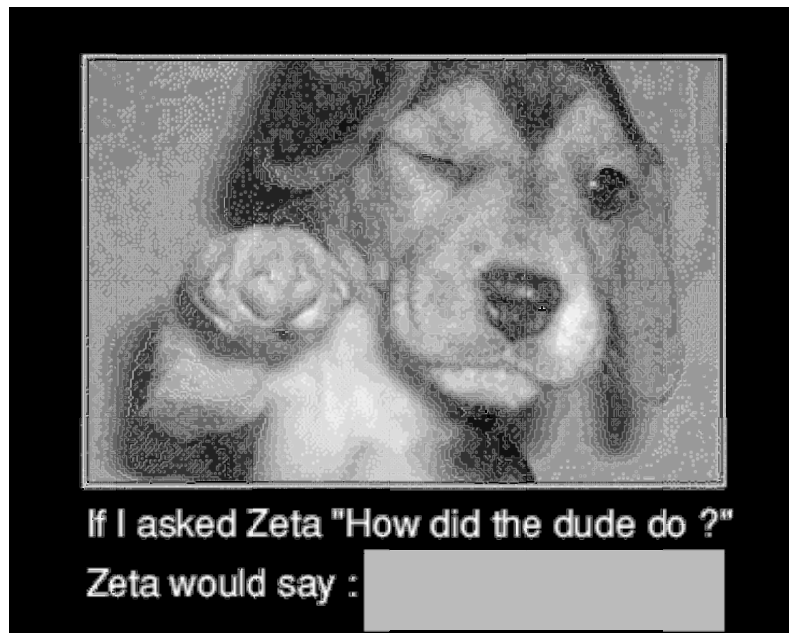
$$(v) \quad \frac{1}{p^5}$$

$$(w) \quad \frac{1}{p^{-5}}$$

$$(x) \quad \frac{(p^5)^5}{(p^6)^6}$$

$$(y) \quad \sqrt{p^{-888}}$$

$$(z) \quad \sqrt{\frac{(p^{-3})^{-11}}{p^{-55}}}$$



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