Lesson 6

GCSE (Year 9) Mathematics Index Form

 $(f) \quad 20^3 \times 10^2$

6.1 The Fundamental Theorem of Arithmetic and Indices

The fundamental theorem of arithmetic says that any[†] positive integer which is not prime can be written as a product of primes.

Examples : (i) $35 = 5 \times 7$ (ii) $12 = 2^2 \times 3$

This idea is the key to answering harder questions involving indices.

6.2 The 8th Law : The Distributive Law

$$35^{3}$$
= $(5 \times 7)^{3}$
= $(5 \times 7) \times (5 \times 7) \times (5 \times 7)$
= $5 \times 5 \times 5 \times 7 \times 7 \times 7$
= $5^{3} \times 7^{3}$
 $\therefore 35^{3} = (5 \times 7)^{3} = 5^{3} \times 7^{3}$

The 8nd Law : The Distributive Law $(a \times b)^m = a^m \times b^m$

6.3 'Together' Questions

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

(a)
$$15^4$$
 (b) 12^5 (c) $6^3 \times 2^2$

(e) $21^4 \times 3^2$

 † Except the number 1.

 $(d) 10^3 \times 2^5$

6.4 Exercise

Question 1

Complete the following tables,

Number	Written as a power of 2	Number	Written as a power of 3
2	2 ¹	3	31
4	2 ²	9	3 ²
	2 ³		3 ³
16			3 ⁴
	25	243	
	26	729	
128		2187	

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

$$(a) \quad 6^{8} \qquad (b) \quad 18^{5} \qquad (c) \quad 12^{7}$$

(d)
$$6^3 \times 2^3$$
 (e) $18^3 \times 3^9$ (f) $\frac{6^5}{2^3}$

(g) 24⁵ (h)
$$128^2 \times 2187^7$$
 (i) $\frac{6^9}{6^2}$

Question 2

Complete the following tables,

Number	Written as a power of 3	Number	Written as a power of 5
3		5	
	3 ²		5 ²
	3 ³		5 ³
81		625	
	35		5 ⁵
729		15625	
2187		78125	

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

(a)
$$15^7$$
 (b) 75^4 (c) 45^8

(d)
$$15^5 \times 3^4$$
 (e) $75^4 \times 3^7$ (f) $\frac{15^8}{3^5}$

(g)	$45^7 \times 5^8$	(h)	$729^3 \times 15625^6$	(i)	$\frac{15^{9}}{15^{4}}$
					154

Question 3

Complete the following tables,

Number	Written as a power of 2	Number	Written as a power of 5
2		5	
	2 ²		5 ²
	2 ³	125	
16			54
	2 ⁵		55
	26	15625	
	27	78125	

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

(a)
$$10^{12}$$
 (b) 50^{14} (c) 20^{18}

(d)
$$10^5 \times 4^3$$
 (e) $50^4 \times 25^4$ (f) $\frac{50^8}{10^5}$

(g)
$$16^{7} \times 10^{8}$$
 (h) $78125^{3} \times 100^{6}$ (i) $\frac{40^{9}}{20^{4}}$

Question 4

(**a**) Write 135 as a product of primes.

(**b**) Hence, or otherwise, write in prime index form, $p^m q^n$, where p and q are prime numbers, the value of;

 $135^{6} \times 15^{5}$

Question 5

(**a**) Write 180 as a product of primes.

(**b**) Hence, or otherwise, write in prime index form, $p^m q^n$, where p and q are prime numbers, the value of;

 $180^{6} \times 6^{8}$

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk