Lesson 6

GCSE (Year 9) Mathematics
Index Form

6.1 The Fundamental Theorem of Arithmetic and Indices

The fundamental theorem of arithmetic says that any† positive integer which is not prime can be written as a product of primes.

Examples : **(i)** $35 = 5 \times 7$
(ii) $12 = 2^2 \times 3$

This idea is the key to answering harder questions involving indices.

6.2 The 8th Law : The Distributive Law

$$\begin{aligned} & 35^3 \\ = \; & (5 \times 7)^3 \\ = \; & (5 \times 7) \times (5 \times 7) \times (5 \times 7) \\ = \; & 5 \times 5 \times 5 \times 7 \times 7 \times 7 \\ = \; & 5^3 \times 7^3 \end{aligned}$$

$$\therefore 35^3 = (5 \times 7)^3 = 5^3 \times 7^3$$

The 8nd Law : The Distributive Law

$$(a \times b)^m = a^m \times b^m$$

6.3 'Together' Questions

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

(a) 15^4 ***(b)*** 12^5 ***(c)*** $6^3 \times 2^2$

(d) $10^3 \times 2^5$ ***(e)*** $21^4 \times 3^2$ ***(f)*** $20^3 \times 10^2$

† Except the number 1.

6.4 Exercise

Question 1
Complete the following tables,

Number	Written as a power of 2
2	2^1
4	2^2
	2^3
16	
	2^5
	2^6
128	

Number	Written as a power of 3
3	3^1
9	3^2
	3^3
	3^4
243	
729	
2187	

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

(a) 6^8 ***(b)*** 18^5 ***(c)*** 12^7

(d) $6^3 \times 2^3$ ***(e)*** $18^3 \times 3^9$ ***(f)*** $\frac{6^5}{2^3}$

(g) 24^5 ***(h)*** $128^2 \times 2187^7$ ***(i)*** $\frac{6^9}{6^2}$

Question 2

Complete the following tables,

Number	Written as a power of 3
3	
	3^2
	3^3
81	
	3^5
729	
2187	

Number	Written as a power of 5
5	
	5^2
	5^3
625	
	5^5
15625	
78125	

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

(a) 15^7 ***(b)*** 75^4 ***(c)*** 45^8

(d) $15^5 \times 3^4$ ***(e)*** $75^4 \times 3^7$ ***(f)*** $\frac{15^8}{3^5}$

(g) $45^7 \times 5^8$ ***(h)*** $729^3 \times 15625^6$ ***(i)*** $\frac{15^9}{15^4}$

Question 3

Complete the following tables,

Number	Written as a power of 2
2	
	2^2
	2^3
16	
	2^5
	2^6
	2^7

Number	Written as a power of 5
5	
	5^2
125	
	5^4
	5^5
15625	
78125	

Write answers in prime index form, $p^m q^n$, where p and q are prime numbers.

(a) 10^{12} ***(b)*** 50^{14} ***(c)*** 20^{18}

(d) $10^5 \times 4^3$ ***(e)*** $50^4 \times 25^4$ ***(f)*** $\dfrac{50^8}{10^5}$

(g) $16^7 \times 10^8$ ***(h)*** $78125^3 \times 100^6$ ***(i)*** $\dfrac{40^9}{20^4}$

Question 4

(a) Write 135 as a product of primes.

(b) Hence, or otherwise, write in prime index form, $p^m q^n$, where p and q are prime numbers, the value of;

$$135^6 \times 15^5$$

Question 5

(a) Write 180 as a product of primes.

(b) Hence, or otherwise, write in prime index form, $p^m q^n$, where p and q are prime numbers, the value of;

$$180^6 \times 6^8$$