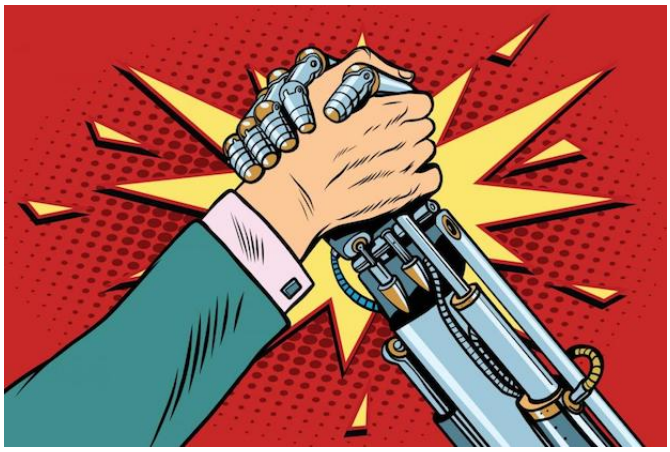
Chapter 2

GCSE and **A-Level Pure Mathematics**
Vectors I

Calculator Needed

2.1 Magnitude and Direction

An interesting aspect of today's questions is that humans and computers would answer them using different methods. To a human, the computer method is long and tedious. To a computer, the human method is flawed: it seemingly can't tell the difference between two possible answers, only one of which is correct.



2.2 Example

Find the magnitude and the direction of the vector, $\boldsymbol{r}$, where

$$\boldsymbol{r} = \begin{pmatrix} 8.4 \\ -3.7 \end{pmatrix}$$

Man v Machine

In answering this question the crucial first step is to draw a diagram. This is because the human solution method uses “short cut” mathematics in which the diagram is used to guide the mathematics.

To draw the diagram keep in mind that the horizontal instruction is the upper number, and the vertical number is the lower number.

Sketch the vector $\boldsymbol{r}$ in the space below;

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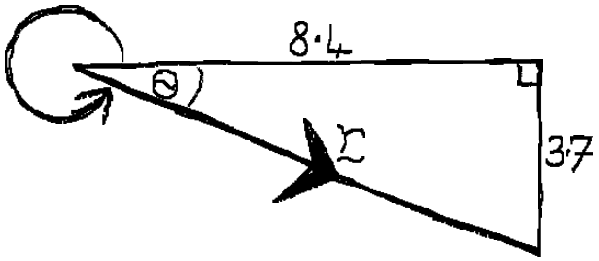
Now turn the page to see if you got it correct.

Here is my sketch of ***r.***
How does your sketch compare with mine ?



Finding the magnitude (the length of the vector) is the easy part of the question.
It's a straight forward use of the Theorem of Pythagoras,

$|\boldsymbol{r}| = \sqrt{8.4^2 + (3.7)^2}$ Notice I used the + 3.7 from the diagram

$= \sqrt{70.56 + 13.69}$

$= \sqrt{84.25}$

$= 9.179$

Now to work out the angle marked θ, again using + 3.7 from the diagram

$$\tan\theta = \frac{3.7}{8.4}$$

$$\theta = \arctan\left(\frac{3.7}{8.4}\right)$$

$$\theta = 23.8°$$

By convention the direction of a vector is given as an angle measured at;

- the tail of the vector,
- from the positive *x*-axis direction,
- turning anticlockwise.

$\therefore$ In this particular question, $Direction = 360° - 23.8°$

$= 336.2°$

This last step is where the diagram totally guided the mathematics.

If one tried to use the – 3.7 with the *tan* function the mathematics cannot tell the difference between

$$\tan\left(\frac{-3.7}{8.4}\right) \text{ and } \tan\left(\frac{3.7}{-8.4}\right)$$

and yet the vectors $\begin{pmatrix} 8.4 \\ -3.7 \end{pmatrix}$ and $\begin{pmatrix} -8.4 \\ 3.7 \end{pmatrix}$ are in totally different directions !

2.1.2 Example

Let $\boldsymbol{s}$ be the following vector :

$$\boldsymbol{s} = \begin{pmatrix} -2.6 \\ -5.1 \end{pmatrix}$$

Find $|\boldsymbol{s}|$ and also the direction in which $\boldsymbol{s}$ acts.

Teaching Video : http://www.NumberWonder.co.uk/v9009/2.mp4

After watching the teaching video, write out an answer to the question below.

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[4 marks]

2.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable.
Make the method used clear.

Marks available : 50

Question 1

Find the magnitude and the direction (with the positive x-axis) of each of the following vectors.

(i) $\boldsymbol{a} = \begin{pmatrix} 11.2 \\ -8.7 \end{pmatrix}$

[4 marks]

(ii) $\boldsymbol{b} = \begin{pmatrix} 7.5 \\ 6 \end{pmatrix}$

[4 marks]

(iii) $\boldsymbol{c} = \begin{pmatrix} -3.5 \\ 8 \end{pmatrix}$

[4 marks]

(iv) $\boldsymbol{d} = \begin{pmatrix} -13 \\ -7.2 \end{pmatrix}$

[4 marks]

(v) $\boldsymbol{e} = \begin{pmatrix} 14.3 \\ -19 \end{pmatrix}$

[4 marks]

Question 2

Find the magnitude and direction (with the positive x-axis) of each of these vectors.

(i) $\boldsymbol{f} = \begin{pmatrix} 7.4 \\ -2.7 \end{pmatrix}$

[4 marks]

(ii) $\boldsymbol{g} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$

[4 marks]

(iii) $\boldsymbol{h} = \begin{pmatrix} -3.5 \\ 3.5 \end{pmatrix}$

[4 marks]

(iv) $\boldsymbol{i} = \begin{pmatrix} -15 \\ -6.2 \end{pmatrix}$

[4 marks]

(v) $\boldsymbol{j} = \begin{pmatrix} 1.3 \\ -16 \end{pmatrix}$

[4 marks]

Question 3

If $\boldsymbol{k} = \boldsymbol{l} + 2\,\boldsymbol{m} + 3\,\boldsymbol{n}$ and

$$\boldsymbol{l} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \qquad \boldsymbol{m} = \begin{pmatrix} 8 \\ 5 \end{pmatrix} \qquad \boldsymbol{n} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

find the magnitude and direction of $\boldsymbol{k}$.

[2 marks]

Question 4

A vector is of magnitude 8.2 cm and acts at an angle of 65°.

(i) Accurately sketch, full size, this vector.

(ii) From the sketch, make measurements, and so write the vector in the form

$$\begin{pmatrix} p \\ q \end{pmatrix}$$

(iii) Use right angled trigonometry (i.e. SOH CAH TOA) to determine the answer to part **(ii)** to an accuracy of four decimal places places.

[4 marks]

Question 5

The vector $\boldsymbol{A}$ is of length 9.3 cm.

$$\boldsymbol{A} = \begin{pmatrix} 6 \\ y \end{pmatrix}$$

Find the two possible values of y and the two possible directions in which this vector is acting.

[2 marks]

Question 6

Write the vector $\boldsymbol{z}$ as a length and a ***bearing.***

$$\boldsymbol{z} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

[2 marks]