**Chapter 4**

**GCSE** and **A-Level Pure Mathematics**
**Vectors I**

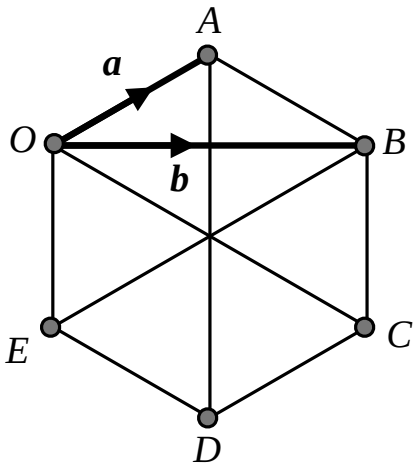
## 4.1 Vector Geometry

Many GCSE vector questions specify a couple of vectors and ask that they be used to move between points on a simple geometric shape such as a triangle, rectangle, hexagon, trapezium or parallelogram.

## 4.2 Example

The diagram, which is not drawn to scale, shows a regular hexagon *OABCDE*

$\overrightarrow{OA} = \boldsymbol{a}$ $\overrightarrow{OB} = \boldsymbol{b}$




Teaching Video : http://www.NumberWonder.co.uk/v9009/4a.mp4
http://www.NumberWonder.co.uk/v9009/4b.mp4

<= Part 1 Part 2 =>

Watch the teaching video, then express the following in terms of ***a*** and ***b***;

**( i )** $\overrightarrow{EB} =$ **( ii )** $\overrightarrow{CD} =$

**( iii )** $\overrightarrow{AB} =$ **( iv )** $\overrightarrow{BC} =$

**( v )** $\overrightarrow{AD} =$ **( vi )** $\overrightarrow{BD} =$

**[ 6 marks ]**

**4.3 Exercise**

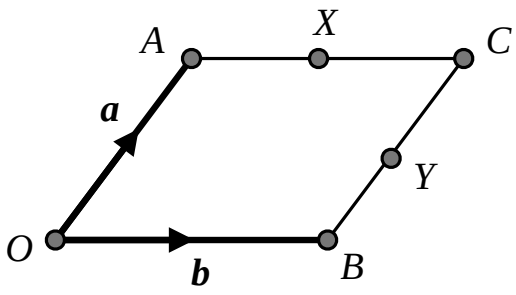
*Any solution based entirely on graphical*
*or numerical methods is not acceptable.*
*Make the method used clear.*
Marks available : 40

**Question 1**

The diagram, which is not drawn to scale, shows a parallelogram *OACB* with

$\overrightarrow{OA} = \boldsymbol{a}$ $\overrightarrow{OB} = \boldsymbol{b}$

The point *X* is the mid-point of *AC* and the point *Y* is the mid-point of *BC*.




**( a )** Express the following vectors in terms of ***a*** and ***b***;

**( i )** $\overrightarrow{BC} =$ **( ii )** $\overrightarrow{AC} =$

**( iii )** $\overrightarrow{BO} =$ **( iv )** $\overrightarrow{CY} =$

**( v )** $\overrightarrow{AB} =$ **( vi )** $\overrightarrow{OX} =$

**( vii )** $\overrightarrow{AX} =$ **( viii )** $\overrightarrow{XY} =$

**[ 8 marks ]**

**( b )** If $|\boldsymbol{a}| = |\boldsymbol{b}|$ which one of of following best describes shape *OACB* ?

**A** triangle **B** trapezium
**C** rhombus **D** cube

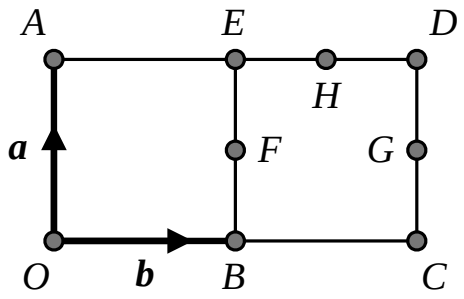
**[ 1 mark ]**

**Question 2**

The diagram, which is not drawn to scale, shows two squares *OBEA* and *BCDE*.

$\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$

The points *F*, *G* and *H* are the mid-points of *BE*, *CD* and *ED* respectively.




**( a )** Express the following vectors in terms of ***a*** and ***b***;

**( i )** $\overrightarrow{BG} =$ **( ii )** $\overrightarrow{GO} =$

**( iii )** $\overrightarrow{AC} =$ **( iv )** $\overrightarrow{GE} =$

**( v )** $\overrightarrow{GA} =$ **( vi )** $\overrightarrow{HO} =$

**[ 6 marks ]**

**( b )** **( i )** Show that $\overrightarrow{AC} = \lambda \overrightarrow{GE}$ for an integer value of $\lambda$ which is to be found.

**[ 1 mark ]**

**( ii )** WWhich one of the following is true about $\overrightarrow{AC}$ and $\overrightarrow{GE}$ ?

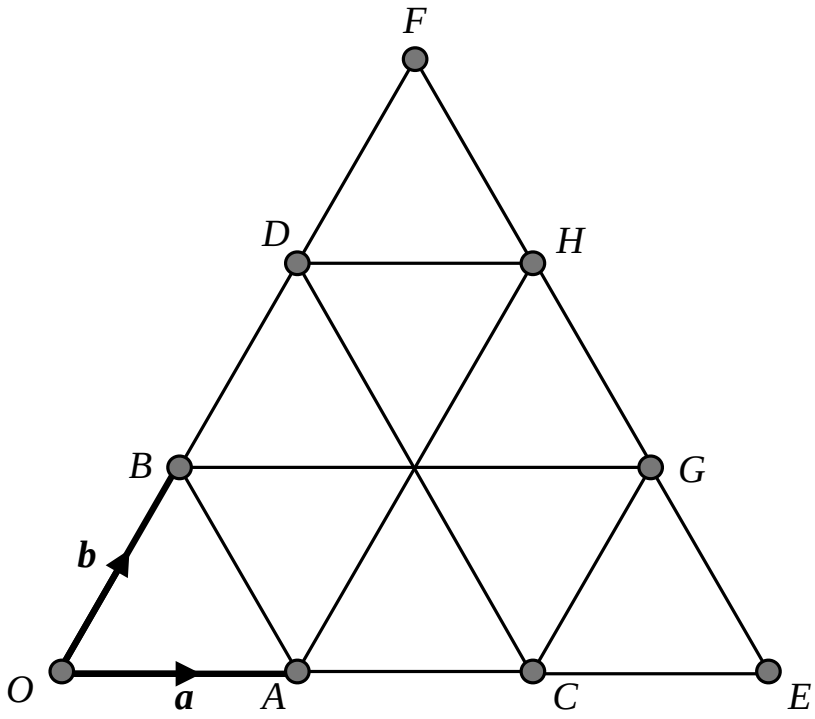
**A** same magnitude **B** same direction

**C** perpendicular **D** parallel

**[ 1 mark ]**

**Question 3**

On the lattice of equilateral triangles, $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$




**( a )** Express the following vectors in terms of ***a*** and ***b***;

**( i )** $\overrightarrow{OH} =$ **( ii )** $\overrightarrow{BE} =$

**( iii )** $\overrightarrow{ED} =$ **( iv )** $\overrightarrow{DA} =$

**[ 4 marks ]**

**( b )** Given that ***a*** and ***b*** are unit vectors, find the magnitude of $\overrightarrow{OG}$

**[ 2 marks ]**

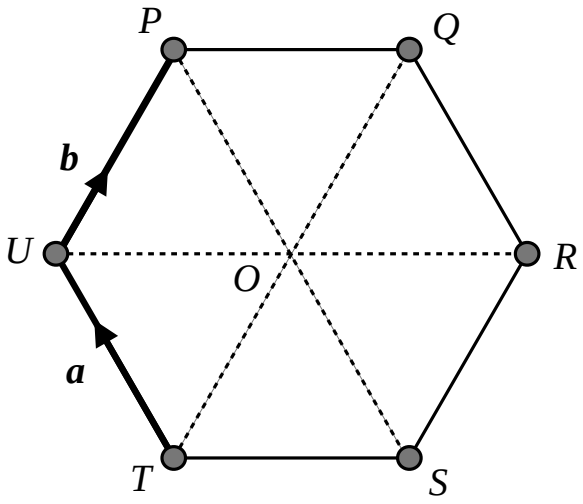
**Question 4**

*GCSE Examination Question, May 2008, 4H, Q21*

*PQRSTU*, which is not drawn to scale, is a regular hexagon, centre *O*

The hexagon is made from six equilateral triangles of side 2.5 cm

$\overrightarrow{TU} = \boldsymbol{a}$ and $\overrightarrow{UP} = \boldsymbol{b}$




**( a )** Find, in terms of ***a*** and ***b***, the vectors

**( i )** $\overrightarrow{TP} =$

**[ 1 mark ]**

**( ii )** $\overrightarrow{PO} =$

**[ 1 mark ]**

**( iii )** $\overrightarrow{UO} =$

**[ 1 mark ]**

**( b )** Find the modulus (magnitude) of $\overrightarrow{UR}$

**[ 1 marks ]**

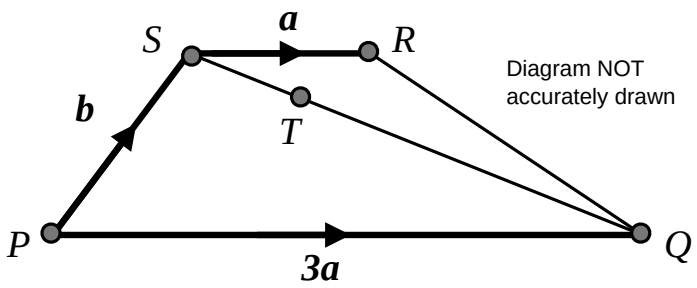
**Question 5**

*GCSE Examination Question, November 2010, 4H, Q21*

*PQRS*, which is not drawn to scale, is a trapezium with *PQ* parallel to *SR*

$\overrightarrow{SR} = \boldsymbol{a}$ $\overrightarrow{PQ} = 3\boldsymbol{a}$ $\overrightarrow{PS} = \boldsymbol{b}$

*T* is the point on *SQ* such that $ST = \frac{1}{4} SQ$




**( a )** Find, in terms of ***a*** and ***b,***

**( i )** $\overrightarrow{PR} =$

**[ 1 mark ]**

**( ii )** $\overrightarrow{SQ} =$

**[ 1 mark ]**

**( iii )** $\overrightarrow{PT} =$

**[ 1 mark ]**

**( b )** $\overrightarrow{PT} = k\,\overrightarrow{PR}$ where *k* is a fraction

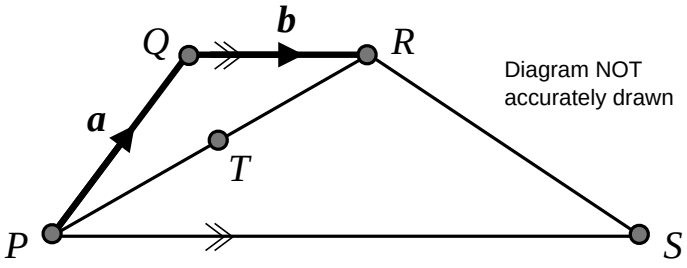
**( i )** What does this result tell you about the points *P*, *T* and *R* ?

**( ii )** Find the value of *k*

**[ 2 marks ]**

**Question 6**

*GCSE Examination Question, June 2011, 4H, Q24 edited*




The diagram shows a trapezium *PQRS,* which is not drawn to scale.

- *T* is the midpoint of *PR*
- *PS* parallel to *QR*
- *PS* = 4 *QR*
- $\overrightarrow{PQ} = \boldsymbol{a}$ and $\overrightarrow{QR} = \boldsymbol{b}$

Find, in terms of ***a*** and ***b,***

**( i )** $\overrightarrow{PS} =$

**[ 1 mark ]**

**( ii )** $\overrightarrow{PR} =$

**[ 1 mark ]**

**( iii )** $\overrightarrow{RS} =$

**[ 1 mark ]**

**( iv )** $\overrightarrow{QT} =$

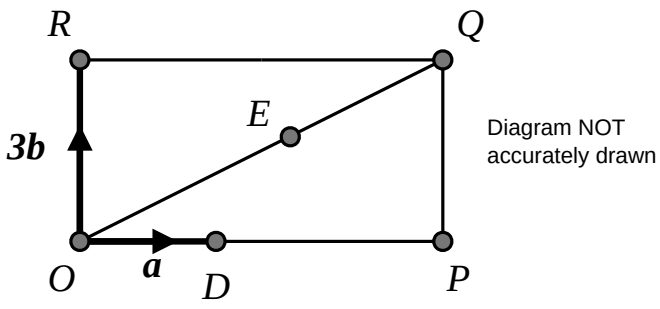
**[ 1 mark ]**

**( v )** $\overrightarrow{TS} =$

**[ 1 mark ]**

**Question 7**

*GCSE Examination Question, January 2012, 4H, Q22*




*OPQR*, which is not drawn to scale, is a rectangle

*D* is the point on *OP* such that $OD = \frac{1}{3} OP$

*E* is the point on *OQ* such that $OE = \frac{2}{3} OQ$

$\overrightarrow{OD} = \boldsymbol{a}$ $\overrightarrow{OR} = 3\boldsymbol{b}$

Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

**( i )** $\overrightarrow{OQ} =$

**[ 1 mark ]**

**( ii )** $\overrightarrow{OE} =$

**[ 1 mark ]**

**( iii )** $\overrightarrow{DE} =$

**[ 1 mark ]**