



**Simultaneous
Equations**

**One Linear
One Quadratic**

Simultaneous Equations

One Linear and One Quadratic

Simultaneous Equations : GCSE

Lesson 1

1.1 Introduction

Previously, we have looked at solving simultaneous equations where both equations were linear. That is, they were *line like*. Geometrically, this equivalent to finding the crossing point of two straight lines.

Now we move on to the situation where one of the two equations is quadratic. This could be a circle, for example;

$$x^2 + y^2 = 64$$

or a parabola, for example;

$$y = x^2 + 2x + 1$$

Here is an example of an examination question;

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Solve the simultaneous equations

$$y = 2x - 3 \quad (\textit{line})$$

$$x^2 + y^2 = 2 \quad (\textit{circle})$$

Geometrically, this is asking for the points (if any) where the line cuts the circle. Typically there will be two points but there could be one or none.

This topic is really five previous topics glued together.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

We'll revise each of these separately, then look at how to glue the bits together.

In an examination, a *solve these simultaneous equations question* is worth about six marks. The bits glued together will also be tested separately elsewhere in the exam. So, being good at this topic is potentially worth about 6% in each of the two papers.

1.2 FOIL

This is a first and crucial step.

If you get this wrong, the mark scheme often states that no marks are to be awarded.

So an error here is really bad: *Go direct to Jail : Do not pass Go, Do not collect £200*

Questions

Expand the brackets;

(i) $(x + 6)(x + 5)$

(ii) $(x + 1)(4x + 7)$

(iii) $(2x + 5)(3x + 4)$

(iv) $(8x + 5)(11x + 9)$

(v) $(x + 8)(x - 3)$

(vi) $(x - 5)(3x + 4)$

(vii) $(x - 7)(5x - 2)$

(viii) $(x + 5)^2$

(ix) $(2x + 7)^2$

(x) $(5x - 3)^2$

The last three questions are the most important.

1.3 Gathering together like terms

Example

Simplify $3x^2 - 3 + 4x + 2x + x^2 + 7$

Try this, then check with the answer at the bottom of the following page.

Questions

Simplify each of the following;

(i) $x^2 + 5x + 7x + 11$

(ii) $2x^2 + 11x - 6x + 3$

(iii) $5x^2 + 4x + 3 + 5x - 2$

(iv) $x^2 + 4x - 7 - 9x + 3$

(v) $9x^2 + 13x - 6x^2 + 3 - x$

(vi) $3x^2 + 14x + (5x - 3) - x^2$

(vii) $5x^2 - 7x - (4x + 3)$

(viii) $x^2 + 7x + (7 - 5x) - 3$

(ix) $8x^2 + 14x - (4x^2 - 3x) - x^2$

(x) $4(3x^2 + 2x) - (7 - 5x) - 1$

1.4 Rearranging equations into the form $f(x) = 0$

Example

Rearrange the following equation into the form $f(x) = 0$

$$5x^2 + 4x + 8 = 2x + 5$$

Try this, then check with the answer at the bottom of the following page.

Questions

Rearrange each of the following equation into the form $f(x) = 0$

(i) $8x^2 + 9x + 5 = 4x + 2$

(ii) $2x^2 + 3x + 11 = 7x + 3$

(iii) $7x^2 - 5x + 1 = 3x - 7$

(iv) $9x^2 + 7x - 8 = 5 - 3x$

Previous page answer: $4x^2 + 6x + 4$

$$(v) \quad 8x^2 + 14x - 8 = 2x^2 - 3x + 2$$

$$(vi) \quad 2x^2 + 15x - 4 = 2x^2 + 3x + 28$$

$$(vii) \quad 5x^2 + (6x - 5) = 9 - 2x + x^2$$

$$(viii) \quad 11x^2 - (4x + 3) = 5x^2 - 15$$

$$(ix) \quad 8x^2 - (7x - 6) = 5 - 3x$$

$$(x) \quad 7x^2 - (3 + 7x) = 2x^2 - (6x - 1)$$

1.5 Mixture

Before tackling the last two items on the introduction's lists, here is some question that mix the first three ideas.

Questions

Rearrange each of the following equation into the form $f(x) = 0$

(i) $(x + 4)(x + 6) = 5x + 3$

(ii) $(3x + 9)(2x + 3) = 8x - 13$

(iii) $(x + 5)(x - 5) = 7x + 13$

$$\text{(iv)} \quad (x + 4)(3x - 5) = 2x^2 - 5$$

$$\text{(v)} \quad (x + 7)^2 = 4x + 7$$

$$\text{(vi)} \quad (2x - 3)^2 = (x + 2)^2$$

Previous page answer: $5x^2 + 2x + 3 = 0$