Lesson 4

4.1 Solving Quadratic Equations

A quadratic equation is an equation of the form,

$$a\,x^2 + bx + c = 0$$

where a, b and c are constants, and x is the variable.

Last lesson, a large number of quadratic expressions where factorised. The ability to factorise a quadratic expression is one of the crucial steps in how the quadratic equations in this lesson may be solved.

Another crucial step in the solution method is an appreciation of the following theorem,

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The Product of Zero Theorem

Given two numbers which have a product of zero,
either the first number or the second number must be zero.

$$\text{If} \quad p \times q = 0$$

$$\text{Then either } \; p = 0 \text{ or } q = 0$$

The word "or" means either $p = 0$ or $q = 0$ or both $p = 0$ and $q = 0$
It is called an "inclusive or".
(FYI : "Exclusive or", *xor*, excludes the possibility that both are zero)

4.2 Example

The Question :

Solve, $x^2 + 6x + 8 = 0$

The Answer :

$$x^2 + 6x + 8 = 0$$

$$(x + 4)(x + 2) = 0 \qquad \text{Factorise}$$

$$\text{Either } x + 4 = 0 \text{ or } x + 2 = 0 \qquad \text{by The Product of Zero Theorem}$$

$$\therefore\; x = -4 \text{ or } x = -2 \qquad \text{These are the "roots" of the quadratic}$$

Notice : There is not one, but two solutions, often called the roots of the quadratic.
In fact there are exactly two solutions; all possible solutions have been found !

There are two values of x that could be substituted back into the quadratic equation $x^2 + 6x + 8 = 0$ that would make it true. Try it and see !

4.3 Exercise

Solve these quadratic equations.

Every solution should use the word “either” and the word “or” at the appropriate step.

Question 1

(i) $x^2 + 8x + 15 = 0$

(ii) $x^2 + 5x + 4 = 0$

(iii) $x^2 + 9x + 14 = 0$

(iv) $x^2 + 9x + 18 = 0$

(v) $x^2 + 15x + 44 = 0$

(vi) $x^2 + 11x + 24 = 0$

(vii) $x^2 + 11x + 30 = 0$

(viii) $x^2 + 12x + 27 = 0$

(ix) $x^2 + 15x + 50 = 0$ **(x)** $x^2 + 12x + 32 = 0$

4.4 Example

The Question :

Solve, $x^2 - 6x - 55 = 0$

The Answer :

$x^2 - 6x - 55 = 0$

$(x - 11)(x + 5) = 0$ Factorise

Either $x - 4 = 0$ or $x + 5 = 0$ by The Product of Zero Theorem

$\therefore\ x = 4$ or $x = -5$ These are the "roots" of the quadratic

Be Careful : By far the most common mistake is to get the signs of the answers wrong; ($x - 11$) is a factor of the quadratic, and $x = 11$ is a root.

4.5 Exercise

Solve these quadratic equations;

Every solution should use the word "either" and the word "or" at the appropriate step.

(i) $x^2 - 5x - 14 = 0$ **(ii)** $x^2 - 4x - 21 = 0$

(iii) $x^2 - 10x - 24 = 0$

(iv) $x^2 - x - 30 = 0$

(v) $x^2 - 6x - 40 = 0$

(vi) $x^2 + 6x - 16 = 0$

(vii) $x^2 + 8x - 33 = 0$

(viii) $x^2 + 5x - 50 = 0$

(ix) $x^2 + 16x - 17 = 0$

(x) $x^2 + 3x - 40 = 0$

Harder Question

(xi) $2x^2 + 32x - 34 = 0$ **HINT** : Immediately divide throughout by 2

Harder Question

(xii) $3x^2 + 9x - 120 = 0$