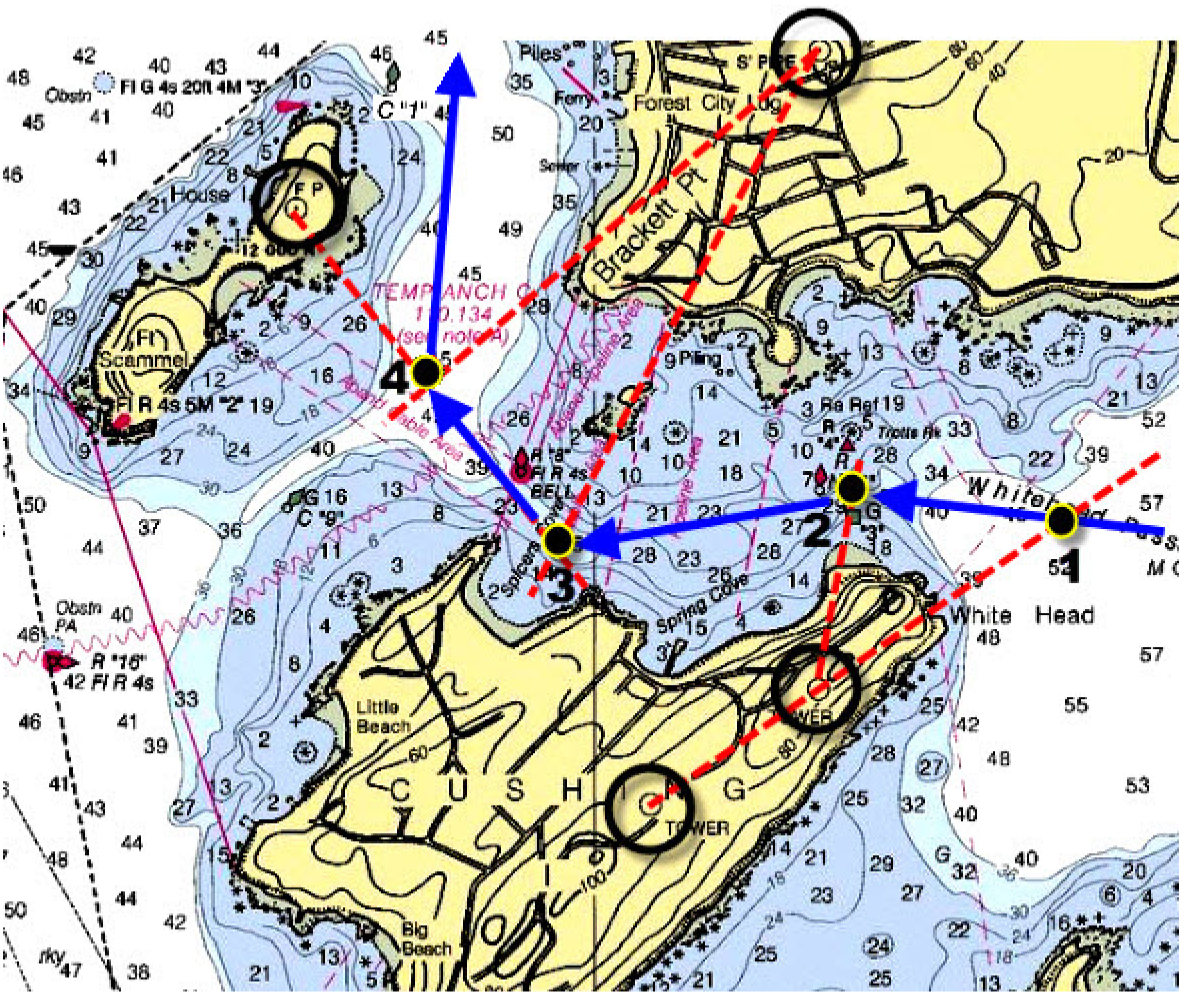
A-Level Pure Mathematics

Year 1 and Year 2

# V E C T O R S
# I I



# V E C T O R S  I I

**Chapter 1**

**A-Level Pure Mathematics**
**Vectors II : Year 1 and Year 2**

## 1.1 Vectors and Kinematics

**Example 1**
A particle moves with initial velocity $( 7\,\boldsymbol{i} + 6\,\boldsymbol{j} )\ \text{ms}^{-1}$
It is accelerating at $( -3\,\boldsymbol{i} + 5\,\boldsymbol{j} )\ \text{ms}^{-2}$
**( i )** What is its velocity when $t = 4$ seconds ?

**( ii )** What is its speed when $t = 4$ seconds ?

**Example 2**
A particle is moving with initial velocity $( -2\,\boldsymbol{i} + \boldsymbol{j} )\ \text{ms}^{-1}$
A constant acceleration of $( \boldsymbol{i} - 2\,\boldsymbol{j} )\ \text{ms}^{-2}$ acts upon it.
**( i )** What is its displacement vector over the next 5 seconds ?

**( ii )** If it was initially at position $( 3\,\boldsymbol{i} + 4\,\boldsymbol{j} )$, where is it when $t$ is 5 seconds ?

## 1.2 Exercise

**Question 1**

A particle is initially moving with velocity ( 3 ***i*** + ***j*** ) $ms^{-1}$

It is constantly accelerating at ( – ***i*** + 2 ***j*** ) $ms^{-2}$

**( i )** What is its velocity when *t* = 7 seconds ?

**( ii )** What is its speed when *t* = 7 seconds ?

**Question 2**

A particle is moving with initial velocity ( 3 ***i*** + 2 ***j*** ) $ms^{-1}$

A constant acceleration of ( 4 ***i*** – **j** ) $ms^{-2}$ acts upon it.

**( i )** What is its displacement vector over the next 3 seconds ?

**( ii )** If initially at position ( – 20 ***i*** + 2 ***j*** ), what is its position when *t* is 3 seconds ?

**Question 3**

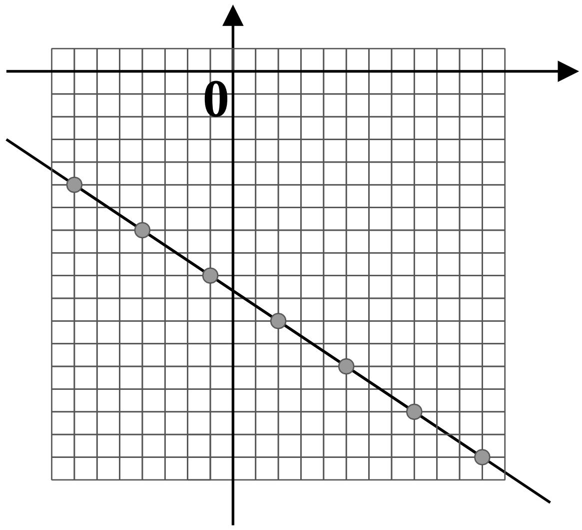
*M1 examination question, May 2010, Q1 with Hint added*

A particle $P$ is moving with constant velocity $(-3\boldsymbol{i} + 2\boldsymbol{j})$ ms$^{-1}$

At time $t = 6$ s $P$ is at the point with position vector $(-4\boldsymbol{i} - 7\boldsymbol{j})$ m

Find the distance of $P$ from the origin at time $t = 2$ s

**HINT :** This diagram may help...




**[ 5 marks ]**

**Question 4**

*M1 examination question, January 2009, Q1*

A particle $P$ moves with constant acceleration $(2\boldsymbol{i} - 5\boldsymbol{j})$ ms$^{-2}$

At time $t = 0$ $P$ has speed $u$ ms$^{-1}$

At time $t = 3$ s, $P$ has velocity $(-6\boldsymbol{i} + \boldsymbol{j})$ ms$^{-1}$

Find the value of $u$

**[ 5 marks ]**

**Question 5**

*M1 examination question, January 2008, Q6*

*[ In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively ]*

A particle $P$ is moving with constant velocity $(-5\,\boldsymbol{i} + 8\,\boldsymbol{j})$ ms$^{-1}$

**( a )** Find the speed of $P$

**[ 2 marks ]**

**( b )** Find the direction of motion of $P$, giving your answer as a bearing

**[ 3 marks ]**

At time $t = 0$ $P$ is at the point $A$ with position vector $(7\,\boldsymbol{i} - 10\,\boldsymbol{j})$ m relative to a fixed origin $O$. When $t = 3$ s, the velocity of $P$ changes and it moves with velocity $(u\,\boldsymbol{i} + v\,\boldsymbol{j})$ ms$^{-1}$, where $u$ and $v$ are constants. After a further 4 s, it passes through $O$ and continues to move with velocity $(u\,\boldsymbol{i} + v\,\boldsymbol{j})$ ms$^{-1}$

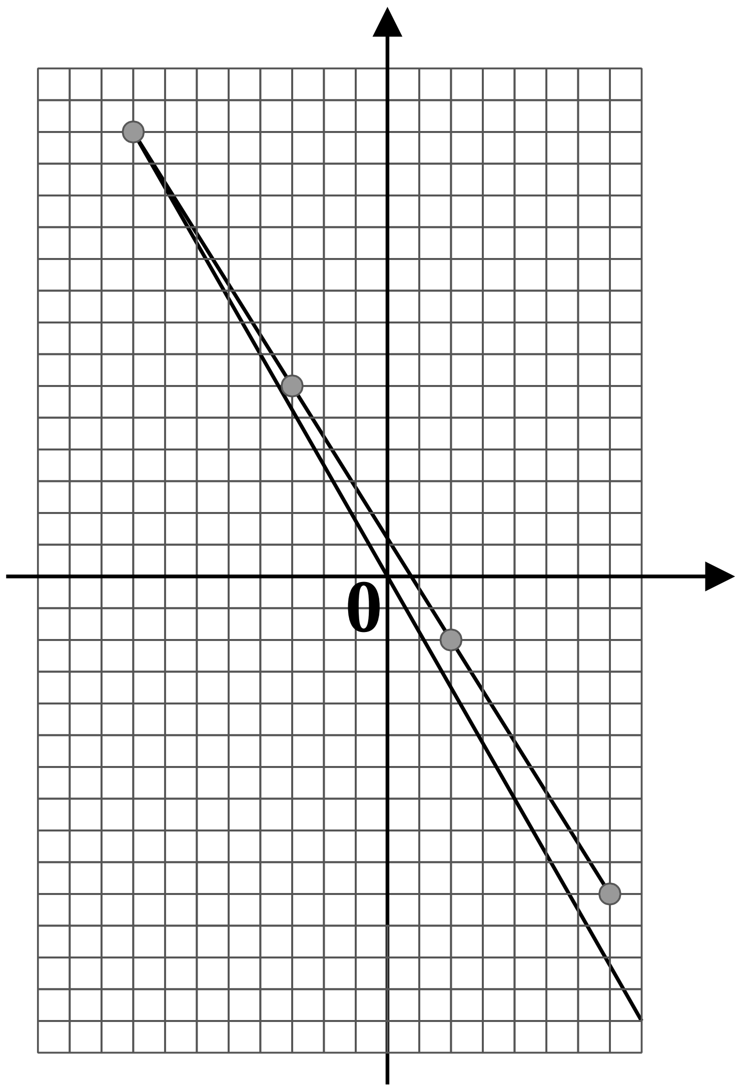
**( c )** Find the values of $u$ and $v$

**[ 5 marks ]**

**( d )** Find the total time taken for $P$ to move from $A$ to a position which is due south of $A$

**[ 3 marks ]**

**HINT :** This diagram may help...



**Question 6**

*M1 examination question, January 2010, Q7*

*[ In this question, the unit vectors **i** and **j** are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin ]*

A ship $S$ is moving along a straight line with constant velocity.
At time $t$ hours the position vector of $S$ is $\boldsymbol{s}$ km
When $t = 0$, $\boldsymbol{s} = 9\,\boldsymbol{i} - 6\,\boldsymbol{j}$
When $t = 4$, $\boldsymbol{s} = 21\,\boldsymbol{i} + 10\,\boldsymbol{j}$

**( a )** Find the speed of $S$

**[ 4 marks ]**

**( b )** Find the direction in which $S$ is moving, giving your answer as a bearing

**[ 2 marks ]**

**( c )** Show that $\boldsymbol{s} = (3t + 9)\,\boldsymbol{i} + (4t - 6)\,\boldsymbol{j}$

**[ 2 marks ]**

A lighthouse $L$ is located at the point with position vector $(18\,\boldsymbol{i} + 6\,\boldsymbol{j})$ km
When $t = T$, the ship $S$ is 10 km from $L$.

**( d )** Find the possible values of $T$.

**[ 6 marks ]**

**Question 7**

*M1 examination question, June 2007, Q7*

A boat $B$ is moving with constant velocity. At noon, $B$ is at the point with position vector ( $3\,\boldsymbol{i} - 4\,\boldsymbol{j}$ ) km with respect to a fixed origin $O$. At 14:30 on the same day, $B$ is at the point with position vector ( $8\,\boldsymbol{i} + 11\,\boldsymbol{j}$ ) km

**( a )** Find the velocity of $b$, giving your answer in the form $p\,\boldsymbol{i} + q\,\boldsymbol{j}$

**[ 3 marks ]**

At time $t$ hours after noon, the position vector of $B$ is $\boldsymbol{b}$ km

**( b )** Find, in terms of $t$, an expression for $\boldsymbol{b}$

**[ 3 marks ]**

Another boat $C$ is also moving with constant velocity. The position vector of $C$, $\boldsymbol{c}$ km, at time $t$ hours after noon, is given by

$$\boldsymbol{c} = ( -9\,\boldsymbol{i} + 20\,\boldsymbol{j} ) + t\,( 6\,\boldsymbol{i} + \lambda\,\boldsymbol{j} )$$

where $\lambda$ is a constant.

Given that $C$ intercepts $B$,

**( c )** find the value of $\lambda$

**[ 5 marks ]**

**( d )** show that, before $C$ intercepts $B$, the boats are moving with the same speed

**[ 3 marks ]**