Chapter 2

A-Level Pure Mathematics
Vectors II : Year 1 and Year 2

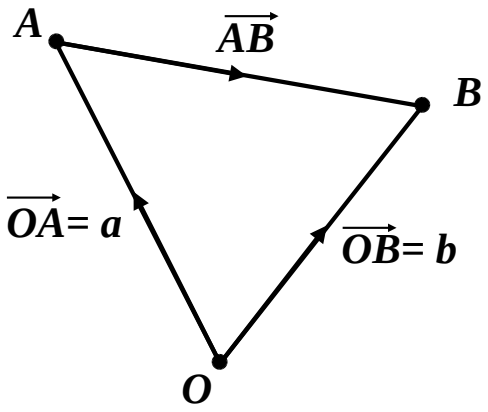
2.1 The Vector Between Two points

Statement :

$$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$$

Proof :

The result is obvious from a study of the following diagram



A more mathematical proof is to argue as follows;

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$$
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} \qquad \square$$

In words we say that $\overrightarrow{AB}$ is $\boldsymbol{b}$ relative to $\boldsymbol{a}$
with the words "relative to" being the interpretation of the minus sign.

i.e. If these were displacement vectors $\overrightarrow{AB}$ is the position of $\boldsymbol{b}$ relative to $\boldsymbol{a}$ which tells you how to get to $\boldsymbol{b}$ from $\boldsymbol{a}$.

2.2 Exercise

Question 1
A is (1, 4)
B is (7, 6)
Write down $\overrightarrow{AB}$

Question 2
C is (– 3, 4)
D is (2, 3)
Write down $\overrightarrow{CD}$

Question 3
P is (7, 3)
Q is (1, 2)
Write down $\overrightarrow{PQ}$

Question 4
A is (– 1, 4)
B is (5, 9)
Write down $\overrightarrow{AB}$

Question 5
A is (– 3, – 5)
B is (2, 1)
Write down $\overrightarrow{BA}$

Question 6
A is (5, – 2)
B is (3, 0)
Write down $\overrightarrow{BA}$

Question 7
P is (4, 1)
Q is (6, 3)
Write down $\overrightarrow{QP}$

Question 8
A is (– 1, – 3)
B is (– 5, – 8)
Write down $\overrightarrow{AB}$

Question 9

P is the point (6, 5) and Q is the point (– 3, 3)

Determine the vector $\overrightarrow{PQ}$

Does your vector take you from P to Q or from Q to P ?

(Draw a sketch of the situation to convince yourself that your answer is correct)

Question 10

M is the point (7, – 4) and N is the point (11, 8)

Determine the vector $\overrightarrow{MN}$

Have you worked out the position of M relative to N or of N relative to M ?

(Draw a sketch of the situation to convince yourself that your answer is correct)

Question 11

C is the point (– 7, 12) and D is the point (8, – 3)

Determine the position of C relative to D

Question 12

At the start of a walk, I am at the position given by $\boldsymbol{r}_{\mathbf{A}} = 1.3\,\boldsymbol{i} + 0.4\,\boldsymbol{j}$ km

I walk directly, in a straight line, to $\boldsymbol{r}_{\mathbf{B}} = 0.3\,\boldsymbol{i} - 0.7\,\boldsymbol{j}$ km

(i) Determine the vector that describes my walk.

(ii) By using the theorem of Pythagoras, and your part (i) answer, determine the distance that I have walked.

Question 13

Two motor boats, *The Dragon*, and *The Runner*, sit side by side upon the ocean. They then separate, each at a constant velocity.

The Dragon has velocity $\boldsymbol{V}_{\mathbf{D}} = 4\,\boldsymbol{i} + 7\,\boldsymbol{j}$ kmh^{-1}

The Runner has velocity $\boldsymbol{V}_{\mathbf{R}} = 5\,\boldsymbol{i} + 5\,\boldsymbol{j}$ kmh^{-1}

(i) Which boat is faster and by how much ?

(ii) Calculate the velocity of *The Dragon* relative to *The Runner*.

(iii) Use your part (ii) answer to determine how long it takes until the two motor boats are 5 km apart.

Question 14

The velocities of particles A and B are $(u\,\boldsymbol{i} - 7\,\boldsymbol{j})$ ms^{-1} and $(5\,\boldsymbol{i} + v\,\boldsymbol{j})$ ms^{-1} respectively. The velocity of A relative to B is $(2\,\boldsymbol{i} - 3\,\boldsymbol{j})$ ms^{-1}

Find the values of u and v.

Question 15

The velocities of two particles A and B are $(13\,\boldsymbol{i} - 3\,\boldsymbol{j})$ ms^{-1} and $(5\,\boldsymbol{i} + 12\,\boldsymbol{j})$ ms^{-1} respectively.

Find;

(i) the speed of B,

(ii) the velocity of B relative to A,

(iii) the angle between this relative velocity and the positive x-axis direction, giving your answer to the nearest degree.

Question 16

I am at the position $\boldsymbol{r} = 7\,\boldsymbol{i} + 5\,\boldsymbol{j}$ m.
My velocity is given by $\boldsymbol{v} = 2\,\boldsymbol{i} + 4\,\boldsymbol{j}$ ms^{-1}
If I have no acceleration, what is my position 4 seconds later ?

Question 17

The position of a particle at time t is given by;

$$\boldsymbol{r} = (2t - 9)\,\boldsymbol{i} + (t - 2)\,\boldsymbol{j}$$

(i) If d is the distance of $\boldsymbol{r}$ from the origin at time t, find an expression for d that involves the square root of a quadratic equation in t.
(HINT : Pythagoras)

(ii) Show, by completing the square on the quadratic, that;

$$\frac{1}{5}d^2 = (t - 4)^2 + 1$$

(iii) What value of t makes $\frac{1}{5}d^2$ as small as possible ?
This is the time at which the particle is closest to the origin.

(iv) What is this minimum distance ?