Chapter 3

3.1 Vectors and Motion

Vectors are used to keep track of shipping and aircraft.
Most importantly, they are used by air traffic control centres to calculate minimum distances between aircraft flight paths, so that 'near misses' are avoided.

Unless told otherwise, ***i*** and ***j*** are **unit** vectors due east and due north respectively.
A **unit** vector has a magnitude of 1.

3.2 The Near Miss Problem

The position of a particle at time t is given by;

$$\boldsymbol{r} = (t - 4)\, \boldsymbol{i} + (t - 2)\, \boldsymbol{j}$$

(i) If d is the distance of $\boldsymbol{r}$ from the origin at time t, find an expression for d that involves the square root of a quadratic equation in t.

(ii) Show, by completing the square on the quadratic that;

$$\frac{1}{2} d^2 = (t - 3)^2 + 1$$

(iii) What value of t makes $\frac{1}{2} d^2$ as small as possible ?

This is the time at which the particle is closest to the origin.

(iv) What is this minimum distance ?

3.3 Exercise

Question 1

Two motor boats *P* and *Q* sit side by side upon the ocean.

At 12.15 pm they separate, each at a constant velocity.

P has velocity $\boldsymbol{V}\mathbf{p} = 8\,\boldsymbol{i} + 7\,\boldsymbol{j}$ ms^{-1} and *Q* has velocity $\boldsymbol{V}\mathbf{q} = 3\,\boldsymbol{i} - 5\,\boldsymbol{j}$ ms^{-1}

(i) Find the velocity of *P* relative to *Q*.

(ii) Use the theorem of Pythagoras, and your part (i) answer to calculate the speed of separation.

(iii) After how many seconds are they 520 metres apart ?

Question 2

I am at the position $\boldsymbol{r} = 16\,\boldsymbol{i} - 7\,\boldsymbol{j}$ m.

My initial velocity is given by $\boldsymbol{u} = 3\,\boldsymbol{i} + \boldsymbol{j}$ $m.s^{-1}$

I am under constant a acceleration of $\boldsymbol{a} = -\,\boldsymbol{i} + \boldsymbol{j}$

(i) Use the formula $\boldsymbol{s} = \boldsymbol{u}\,t + \frac{1}{2}\,\boldsymbol{a}\,t^2$ to find my position 3 seconds later.

(ii) Find an expression (in terms of *t*) for the velocity at time *t*.
HINT: $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}\,t$

(iii) By using your part (ii) answer, or otherwise, find the time at which I am moving in the direction $\boldsymbol{i} + \boldsymbol{j}$

This next question is like the 'near miss' problem looked at previously

Question 3

The position of Cedric, my cyber-dog, at time t is given by;

$$\boldsymbol{r} = (t - 5)\, \boldsymbol{i} + (t + 1)\, \boldsymbol{j} \quad \text{metres.}$$

(i) What is the distance of Cedric from the origin when $t = 5$?

(ii) Find an expression involving t for the distance that Cedric is from the origin at time t.

If Cedric moves further than 30 metres from the origin I can no longer receive his video signal transmission.

(iii) By using your part (ii) answer, or otherwise, find the time at which I lose video signal contact with Cedric.

Question 4

At noon the position of a ship relative to a radar station *O* is (100 ***i*** +60 ***j***) km ***i*** and ***j*** being vectors due east and due north respectively.

The velocity of the ship is constant at $-10\,\boldsymbol{i} + 8\,\boldsymbol{j}$ kmh^{-1}

(i) Find the time when the ship is due north of *O*

(ii) How far the ship then is from *O*

(iii) How far the ship is from *O* at 1 pm

Question 5

Two ice skaters pass close to each other at constant velocities at an ice rink.

Skater *A* has velocity $\boldsymbol{V}_A = \boldsymbol{i} + 3\boldsymbol{j}$ and skater *B* has velocity $\boldsymbol{V}_B = 2\boldsymbol{i} + \boldsymbol{j}$

(i) Find the velocity of *B* relative to *A*.

At $t = 0$ the positions of the skaters are $\boldsymbol{S}_A = \mathbf{i} - 3\mathbf{j}$ and $\boldsymbol{S}_B = -3\boldsymbol{i} + 5\boldsymbol{j}$

(ii) Show that, at time *t*, *A* has position

$$\boldsymbol{R}_A = (1 + t)\boldsymbol{i} + (-3 + 3t)\boldsymbol{j}$$

and find a similar expression for the position of *B* at time *t*.

(iii) Find an expression for the position of *B* relative to *A*.

(iv) By considering your part (iii) expression find the value of *t* for which *A* and *B* were closest together.

(v) What is this closest distance ?