$\sqrt{S \cdot U \cdot R \cdot D \cdot S}$

&

$I^{i} \cdot N^{n} \cdot D^{d} \cdot I^{i} \cdot C^{c} \cdot E^{e} \cdot S^{s}$

Surds and Indices

Lesson 1

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

1.1 Square Roots without a calculator

Example #1
Without using a calculator, find $\sqrt{2704}$

[3 marks]

Example #2
The following number is too big for my calculator;

$$11^{102} \times 13^{284}$$

Even so, square root this number, writing the answer in index form

[2 marks]

1.2 Two Quick Questions

(i) Without using a calculator, making your method clear, determine,

$$\sqrt{3969}$$

[3 marks]

(ii) The following number is too big for my calculator;

$$17^{38} \times 19^{74}$$

Even so, square root this number, writing the answer in index form

[2 marks]

1.3 Square Free

Any number which is not prime can be written as a unique product of primes.
For example,

$$120 = 2^3 \times 3 \times 5$$

Mathematicians talk of the *decomposition* of 120 into a product of primes.

There is another decomposition of 120 that is useful. It revolves around identifying the biggest square number that will divide into 120 exactly.

Reminder: Square Numbers = { 1, 4, 9, 16, 25, 36, ... }

As 4 is the biggest square number that divides into 120 we can write,

$$120 = 4 \times 30$$

Notice that no square number, other than 1, will divide into 30.
Thus 30 is termed square-free or, more succinctly, □ Free.

In summary, our new decomposition takes an integer that is not □ Free and expresses it as a square number multiplied by a square free number.
i.e.

$$Not\ \square\ Free = \square \times \square\ Free$$

Example : Without using a calculator, write $\sqrt{120}$ in the form $a\sqrt{p}$
where a and p are integers
and p is □ FREE.

[2 marks]

1.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 26

Question 1

Without using a calculator, write each of the following in the form $a\sqrt{p}$

where a and p are integers

and p is □ FREE.

(i) $\sqrt{8}$

(ii) $\sqrt{3^3}$

(iii) $\sqrt{48}$

(iv) $\sqrt{98}$

(v) $\sqrt{2^3 \times 7}$

(vi) $\sqrt{2 \times 11^2}$

(vii) $\sqrt{2^8 \times 5}$

[7 marks]

Question 2

Which of the following are ☐ FREE ?

(i) 5 **(ii)** 5^2 **(iii)** 5^3

(iv) 5^4 **(v)** 2×3 **(vi)** $2^3 \times 3$

[2 marks]

Question 3

The following number is too big for my calculator;

$$5^{52} \times 7 \times 13^{95}$$

Even so, square root this number, writing the answer in in the form $a\sqrt{p}$

where a and p are integers, that may be written in index form

and p is ☐ FREE.

[3 marks]

Question 4

Without using a calculator, write each of the following in the form $a\sqrt{p}$

where a and p are integers

and p is ☐ FREE.

(i) $\sqrt{504}$ **(ii)** $\sqrt{1452}$

(iii) $\sqrt{2 \times 3^3 \times 5^3}$ **(iv)** $\sqrt{2 \times 3^5 \times 11}$

[2, 3, 2, 3 marks]

Question 5

Without using a calculator, determine the cube root of 1728

i.e. $\sqrt[3]{1728}$

[4 marks]