Lesson 4

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

4.1 Surds from Quadratics

When solving quadratic equations, the exact answers can be numbers of the form,

$$x = a + b\sqrt{c}$$

A consequence of this is that the arithmetic of such numbers becomes of interest.

Example #1

Find the exact solutions of the quadratic equation $x^2 - 2x - 11 = 0$

[3 marks]

Example #2

Expand the brackets and simplify; $\left(7 + 2\sqrt{5}\right)\left(4 + 3\sqrt{5}\right)$

[3 marks]

4.2 You Try

Expand the brackets, $\left(3 + \sqrt{7}\right)\left(2 + \sqrt{7}\right)$

Once done, check your answer with mine, over the page.

[2 marks]

4.3 You Try Answer

$$(3+\sqrt{7})(2+\sqrt{7}) = 6 + 3\sqrt{7} + 2\sqrt{7} + 7$$
$$= 13 + 5\sqrt{7}$$

[2 marks]

4.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Do NOT use a calculator

Question 1

Expand the brackets and simplify each of the following;

(i) $(6+\sqrt{2})(3+\sqrt{2})$

(ii) $(5+\sqrt{13})(4+\sqrt{13})$

(iii) $(7+3\sqrt{2})(5+\sqrt{2})$

(iv) $(2+\sqrt{3})(1+5\sqrt{3})$

[8 marks]

Question 2

Find the exact solutions of the quadratic equation $x^2 - 2x - 1 = 0$

[3 marks]

Question 3

Expand the brackets and simplify each of the following;

(i) $\left(4 + 7\sqrt{3}\right)\left(5 + 2\sqrt{3}\right)$

(ii) $\left(7 + 3\sqrt{2}\right)^2$

(iii) $\left(6 + \sqrt{5}\right)\left(3 - 2\sqrt{5}\right)$

(iv) $\left(6 - 5\sqrt{3}\right)^2$

[12 marks]

Question 4

Express $\sqrt{80} + \frac{30}{\sqrt{5}}$ in the form $c\sqrt{5}$ where c is an integer

[2 marks]

Question 5

Express $\sqrt{50} + \sqrt{3} \times \sqrt{6} - \frac{14}{\sqrt{2}}$ in as simple a form as possible.

[3 marks]

Question 6

By using the result "a difference of two squares", or otherwise, simplify;

(i) $\left(20 + 3\sqrt{7}\right)\left(20 - 3\sqrt{7}\right)$

[2 marks]

(ii) $\left(11 + 2\sqrt{3}\right)\left(11 - 2\sqrt{3}\right)$

[2 marks]

Question 7

Expand the brackets and simplify;

(i) $(x + 1) (x + 2) (x + 3)$

[3 marks]

(ii) $(x + 2) (x + 5) (x - 3)$

[3 marks]

Question 8

Expand the brackets and simplify;

(i) $\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\left(3+\sqrt{5}\right)$

[4 marks]

(ii) $\left(4+\sqrt{2}\right)\left(1+\sqrt{2}\right)\left(5-\sqrt{2}\right)$

[4 marks]

Question 9

Expand the brackets and simplify,

$\left(1 + 4\sqrt{3}\right)\left(5 + \sqrt{3}\right)\left(2 + 5\sqrt{3}\right)$

[4 marks]

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