Lesson 5

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

5.1 Rationalising the Denominator #2

Two observations, made previously, are these;

Firstly, when solving quadratic equations, numbers of the form $a + b\sqrt{c}$ are often the result. Secondly, it was noted that mathematicians' consider any occurrence of a square root in a denominator undesirable.

These two observations collide when asked to rationalise the denominator of an expression of the form

$$\frac{m}{a + b\sqrt{c}} \text{ where } m, a, b, \text{ and } c \text{ are integers, } c > 0$$

5.2 The Key Idea

The previous tactic of multiplying both numerator and denominator by something that clears the square root from the denominator is still the correct strategy, but what might that something be ?

It's not as straight forward as before as the following attempt using $\sqrt{c}$ shows;

$$\frac{m}{a + b\sqrt{c}} = \frac{m}{a + b\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}}$$

$$= \frac{m\sqrt{c}}{a\sqrt{c} + bc} \text{ which still has a square root in the denominator.}$$

The key idea is to set up a difference of two squares.

The Rationaliser Rule

To rationalise the denominator of

$$\frac{m}{(a \pm b\sqrt{c})} \text{ where } m, a, b, \text{ and } c \text{ are integers, } c > 0$$

multiply both numerator and denominator by

$$(a \mp b\sqrt{c})$$

Proof

$$\frac{m}{(a \pm b\sqrt{c})} = \frac{m}{(a \pm b\sqrt{c})} \times \frac{(a \mp b\sqrt{c})}{(a \mp b\sqrt{c})}$$

$$= \frac{m(a - b\sqrt{c})}{a^2 - b^2 c}$$

Which has no square roots in the denominator. □

5.3 Example

Consider the fraction $\dfrac{21}{5 + 3\sqrt{2}}$ which has a square root in its denominator.

(i) Expand the brackets, $(5 + 3\sqrt{2})(5 - 3\sqrt{2})$

[2 marks]

(ii) Hence, or otherwise, rationalise the denominator of $\dfrac{21}{5 + 3\sqrt{2}}$

[3 marks]

Note the technique of NOT multiplying out the numerator until first cancelling it against the denominator thus avoiding large number mental arithmetic.

5.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 45

Do NOT use a calculator

Question 1

This question is about rationalising the denominator of

$$\frac{12}{3 + \sqrt{5}}$$

Step 1 : Expand the brackets : $\left(3 + \sqrt{5}\right)\left(3 - \sqrt{5}\right)$

[2 marks]

Step 2: Multiply both numerator and denominator by $3 - \sqrt{5}$
That is, calculate and simplify,

$$\frac{12}{(3 + \sqrt{5})} \times \frac{(3 - \sqrt{5})}{(3 - \sqrt{5})}$$

[3 marks]

Question 2

This question is about rationalising the denominator of

$$\frac{132}{9 + 4\sqrt{3}}$$

Step 1 : Expand the brackets : $\left(9 + 4\sqrt{3}\right)\left(9 - 4\sqrt{3}\right)$

[2 marks]

Step 2 : Multiply both numerator and denominator by $9 - 4\sqrt{3}$

That is, calculate and simplify,

$$\frac{132}{(9 + 4\sqrt{3})} \times \frac{(9 - 4\sqrt{3})}{(9 - 4\sqrt{3})}$$

[3 marks]

Question 3

Rationalise the denominator of, $\frac{6}{2 + \sqrt{2}}$

[4 marks]

Question 4
This question is about rationalising the denominator of

$$\frac{60}{7 + 3\sqrt{5}}$$

Step 1 : Expand the brackets : $(7 + 3\sqrt{5})(7 - 3\sqrt{5})$

[2 marks]

Step 2 : Multiply both numerator and denominator by $7 - 3\sqrt{5}$

[3 marks]

Question 5
Rationalise the denominator, writing your answer in the form $a + b\sqrt{c}$

$$\frac{\sqrt{7}}{8 - 3\sqrt{7}}$$

[4 marks]

Question 6

(i) Expand the brackets : $\left(7 + 4\sqrt{3}\right)^2$

[2 marks]

(ii) Expand the brackets : $\left(7 + 4\sqrt{3}\right)\left(7 - 4\sqrt{3}\right)$

[2 marks]

(iii) Hence, rationalise the denominator of $\dfrac{7 + 4\sqrt{3}}{7 - 4\sqrt{3}}$

[2 marks]

Question 7

GCSE Examination Question, June 2018, Q21 (b)

Express $\dfrac{2}{\sqrt{3} - 1}$ in the form $p + \sqrt{q}$ where p and q are integers

Show your working clearly.

[2 marks]

Question 8

Rationalise the denominator of, $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

[4 marks]

Question 9

Rationalise the denominator of $\dfrac{100\sqrt{5}}{7 + 2\sqrt{11}}$

Write your answer in the form $a\sqrt{5} + b\sqrt{55}$

[5 marks]

Question 10

Rationalise the denominator of $\dfrac{46}{5\sqrt{2} + 3\sqrt{3}}$

Write your answer in the form $a\sqrt{2} + b\sqrt{3}$

[5 marks]