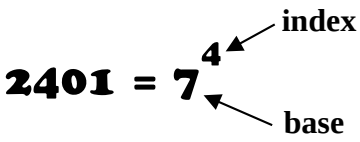
Lesson 7

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

7.1 Index Manipulations



7.2 Introductory Examples

Without using a calculator, write down the values of

(i) $25^{\frac{1}{2}}$ **(ii)** $27^{\frac{1}{3}}$ **(iii)** 3^{-2} **(iv)** $4^{\frac{3}{2}}$ (v) $9^{-\frac{1}{2}}$

(vi) $100^{-\frac{3}{2}}$ **(vii)** $81^{\frac{1}{4}}$ **(viii)** $81^{\frac{3}{4}}$ **(ix)** $81^{-\frac{1}{2}}$ **(x)** 81^{0}

[5 marks]

7.3 Examples Of Tricky Index Questions

Example #1

Given that $81\sqrt{3} = 3^{a}$ find the value of the real valued constant a

[2 marks]

Example #2

Simplify $\left(27x^{12}\right)^{\frac{2}{3}}$

[2 marks]

Example #3

Without using a calculator, and making your method clear, find the square root of

$$2^5 \times 3^4 \times 5^3$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[3 marks]

Example #4

Simplify, $\left(\frac{3x^2}{4\sqrt{y}} \right)^{-3}$

[3 marks]

7.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 45

Do NOT use a calculator

Question 1

Simplify, $(2h)^3$

[2 marks]

Question 2

Simplify, $\left(8x^3\sqrt{y}\right)^2$

[3 marks]

Question 3

Simplify, $\left(\frac{3m^2}{5n} \right)^3$

[3 marks]

Question 4

Simplify, $\left(\frac{a}{b} \right)^{-1}$

[1 mark]

Question 5

Simplify, $\left(\frac{3\sqrt{x}}{4y^5} \right)^{-2}$

[3 marks]

Question 6

Without using a calculator, and making your method clear, find the square root of

$$2^7 \times 3 \times 5^4$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[3 marks]

Question 7

Given that, $8\sqrt{2} = 2^a$ find the value of the real constant a

[2 marks]

Question 8

Simplify, $\left(25x^{12}\right)^{\frac{3}{2}}$

[2 marks]

Question 9

GCSE Examination Question, May 2016, Paper 3H, Q7 (d)

Simplify fully, $\dfrac{28x^5y^3}{4xy^2}$

[3 marks]

Question 10

GCSE Examination Question, January 2017, Paper 4H, Q16

$g = 2^3 \times 3 \times 7^2$ $\qquad$ $h = 2 \times 3 \times 7^3$

(a) Express gh as a product of powers of its prime factors
Simplify your answer

[2 marks]

(b) Find the value of a, the value of b and the value of c given that,

$$\frac{g}{h} = 2^a \times 3^b \times 7^c$$

[3 marks]

(c) Show that, $\left(7 - 2\sqrt{5}\right)\left(7 + 2\sqrt{5}\right) = 29$
Show your working clearly.

[3 marks]

(d) Work out the exact value of n, given that, $\dfrac{1}{\sqrt[3]{9^4}} = 3^n$

[3 marks]

Question 11

GCSE Examination Question, June 2016, Paper 4H, Q15 (b)

Simplify, $\left(8\,a^9\,e^6\right)^{\frac{1}{3}}$

[2 marks]

Question 12

GCSE Examination Question, May 2019, Paper 1H, Q10

$$A = 2^n \times 3 \times 5^m$$

Write 8*A* as a product of powers of its prime factors.

[2 marks]

Question 13

GCSE Examination Question, May 2019, Paper 1H, Q12(c)

Write the following in the form y^b where *b* is a fraction

Write $\frac{\sqrt[4]{y}}{y}$ in the form y^b where *b* is a fraction

[2 marks]

Question 14

GCSE Examination Question, January 2017, Paper 3H(R), Q11 (c)

Simplify, $\left(\frac{y^5}{8\,x^6\,y^8}\right)^{-\frac{1}{3}}$

[3 marks]

Question 15

GCSE Examination Question, January 2017, Paper 3H(R), Q18

Given that *p* is a prime number, rationalise the denominator of

$$\frac{7\sqrt{p} - p^2}{\sqrt{p^3}}$$

Simplify your answer.

[3 marks]

Question 16

GCSE Examination Question, January 2017, Paper 4H(R), Q20

(a) $\left(3 + \sqrt{c}\right)\left(2\sqrt{c} - 3\right) = 1 + k\sqrt{c}$ where c and k are prime numbers.

Find the value of c and the value of k.

[3 marks]

(b)

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$

Find the value of m.

[3 marks]