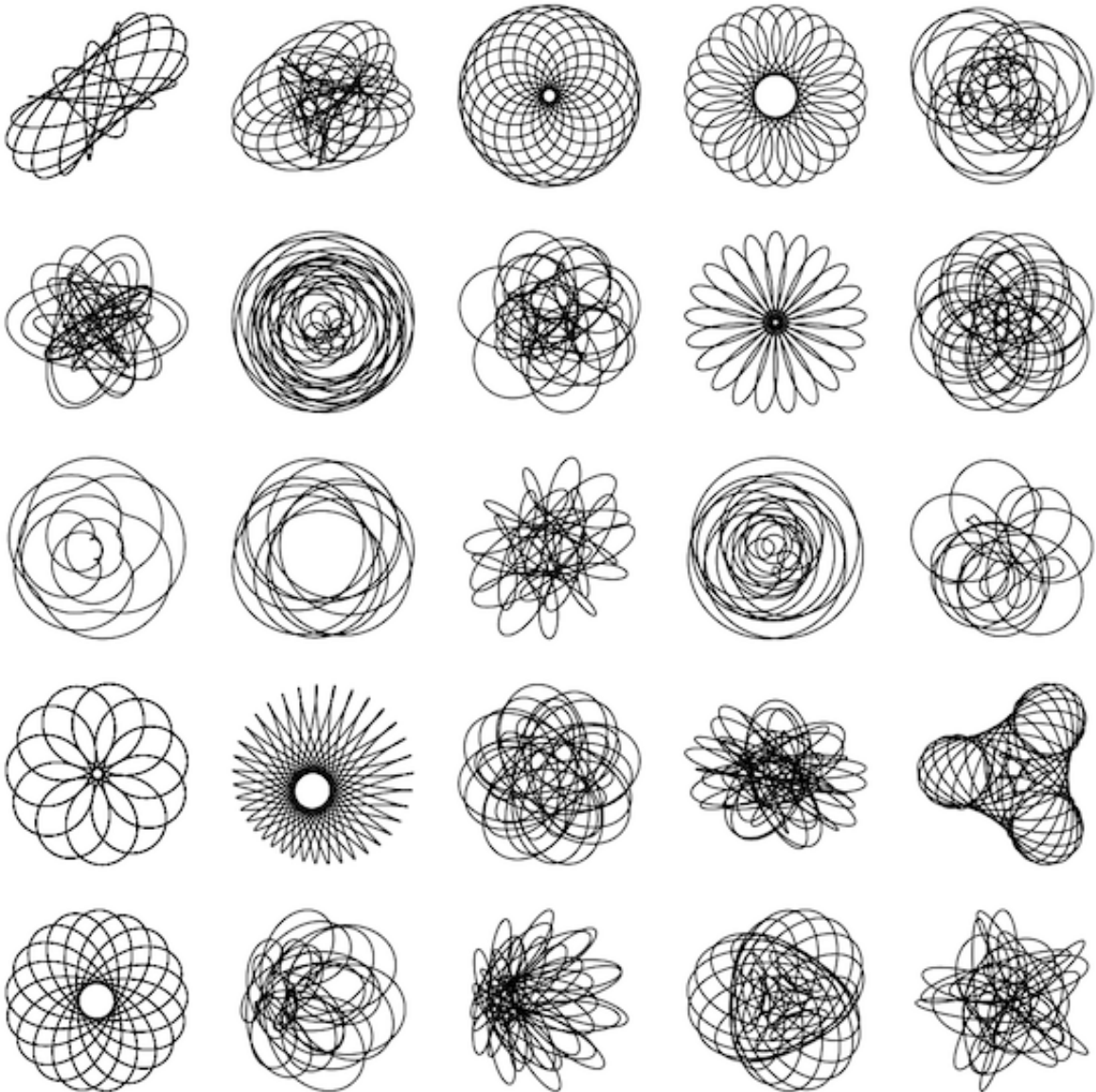


A-Level Pure Mathematics
Year 2

DIFFERENTIATION

III



Chain Rule • Product Rule • Quotient Rule

1.1 The Chain Rule

The Chain Rule is a rule about differentiation that has similarities with the familiar Differentiation Power Rule for differentiating x raised to the power n .

The Differentiation Power Rule

If $y = x^n$ then $\frac{dy}{dx} = n x^{n-1}$ where x is a variable and n is a constant.

1.2 Example

Find the derivative of $y = (4x + 5)^2$ by,

- (i) first expanding the brackets, then using “The Differentiation of x^n Rule”
- (ii) using the “The Chain Rule”

Answer (i)

$$\begin{aligned}y &= (4x + 5)^2 \\y &= (4x + 5)(4x + 5) \\y &= 16x^2 + 40x + 25 \\ \frac{dy}{dx} &= 32x + 40 \\ \frac{dy}{dx} &= 8(4x + 5)\end{aligned}$$

Answer (ii)

Teaching Video : <http://www.NumberWonder.co.uk/v9028/1.mp4>



After watching the video write out the chain rule method of solution here



[2 marks]

1.3 Three For You To Do

Here is the formal statement of The Chain Rule that was given in the video,

The Chain Rule for $y = [f(x)]^n$

$$\text{If } y = [f(x)]^n \text{ then } \frac{dy}{dx} = n [f(x)]^{n-1} f'(x)$$

Now try to apply The Chain Rule to these three further examples. The answers are on the following page so you can immediately check to see if you are correct.

Try 1 $y = (4x + 5)^{20}$

[2 marks]

Try 2 $y = (8x + 1)^5$

[2 marks]

Try 3 $y = (4x^3 + 8)^4$

[2 marks]

Answer to Try 1

$$y = (4x + 5)^{20}$$
$$\frac{dy}{dx} = 20(4x + 5)^{19} \times 4$$
$$\frac{dy}{dx} = 80(4x + 5)^{19}$$

Answer to Try 2

$$y = (8x + 1)^5$$
$$\frac{dy}{dx} = 5(8x + 1)^4 \times 8$$
$$\frac{dy}{dx} = 40(8x + 1)^4$$

Answer to Try 3

$$y = (4x^3 + 8)^4$$
$$\frac{dy}{dx} = 4(4x^3 + 8)^3 \times (12x^2)$$
$$\frac{dy}{dx} = 48x^2(4x^3 + 8)^3$$

1.4 Exercise

Marks Available : 35

Question 1

Differentiate $y = (5x^2 + 7)^3$

[2 marks]

Question 2

Differentiate $y = 5(11 - 6x^2)^5$

[2 marks]

Question 3

Differentiate $y = \sqrt{9 - 5x}$

[2 marks]

Question 4

Differentiate the following function,

$$f(x) = 4(9 + 14x)^{\frac{3}{2}}$$

[2 marks]

Question 5

Differentiate the following function,

$$f(x) = \frac{5}{(7x + 8)^3}$$

[3 marks]

Question 6

Find $\frac{dy}{dx}$ when $y = \frac{5}{3(7-2x)^5}$

[3 marks]

Question 7

Differentiate the following function;

$$y = 6 + \frac{1}{(3x^2 + 2)}$$

[3 marks]

Question 8

Consider the curve,

$$y = (2x - 3)^4$$

Find the equation of the tangent to the curve at the point (2, 1)

Give your answer in the form $y = mx + c$

[3 marks]

Question 9

(a) A curve has the following equation,

$$y = (x + 3)^3 - 4(x + 3)$$

Find the coordinates of points on the curve with gradient 8.

[3 marks]

(b) Repeat part (a) for the curve with the following equation,

$$y = x^3 - 4x$$

[3 marks]

(c) How are your part (a) and part (b) answers related ?

[1 mark]

Question 10

- (a) Find the equation of the tangent to the curve,

$$y = \frac{2}{x^2 - 3}$$

at the point $\left(3, \frac{1}{3} \right)$

Give your answer in the form $ax + by + c = 0$

[4 marks]

- (b) Find the normal to the curve at the same point.
Again, give your answer in the form $ax + by + c = 0$.

[4 marks]