**Lesson 13**

**A-Level Pure Mathematics : Year 2**

**Differentiation III**

## 13.1 Later Date Revision

Marks Available : 40

**Table of Standard Derivatives**

| $f(x)$ | $f'(x)$ | In Formula Book ? |
|---|---|---|
| $x^n$ | $n\,x^{n-1}$ | No |
| $e^x$ | $e^x$ | No |
| $\ln x$ | $\frac{1}{x}$ | No |
| $\sin x$ | $\cos x$ | No |
| $\cos x$ | $-\sin x$ | No |
| $\tan x$ | $\sec^2 x$ | Yes |
| $\csc x$ | $-\csc x \cot x$ | Yes |
| $\sec x$ | $\sec x \tan x$ | Yes |
| $\cot x$ | $-\csc^2 x$ | Yes |
| $\arcsin x$ | $\frac{1}{\sqrt{1-x^2}}$ | Yes |
| $\arccos x$ | $-\frac{1}{\sqrt{1-x^2}}$ | Yes |
| $\arctan x$ | $\frac{1}{1+x^2}$ | Yes |

**Question 1**

Show that the derivative with respect to $x$ of

$$y = \sec x \tan x$$

is

$$\frac{dy}{dx} = \sec x \left( 2 \sec^2 x - 1 \right)$$

**[ 4 marks ]**

**Question 2**

Show that the derivative with respect to $x$ of;

$$y = \csc x \cot x$$

is

$$\frac{dy}{dx} = \csc x \left( 1 - 2 \csc^2 x \right)$$

**[ 4 marks ]**

**Question 3**

Consider the function;

$$f(x) = \frac{8}{(1 - 3x)^3}$$

Show that;

$$f'(1) = \frac{9}{2}$$

**[ 4 marks ]**

**Question 4**

*A-Level Examination Question from January 2009, Paper C3 (Edexcel)*

Find the equation of the tangent to the curve

$$x = \cos(2y + \pi) \quad \text{at} \quad \left(0, \frac{\pi}{4}\right)$$

Give your answer in the form $y = ax + b$, where $a$ and $b$ are constants to be found.

**[ 6 marks ]**

**Question 5**

The curve

$$y = ln\left(x^2 - 3\right)$$

crosses the $x$-axis at $A$ and $B$.

**( i )** Find the coordinates of $A$ and $B$

**[ 3 marks ]**

**( ii )** The normals at $A$ and $B$ meet at $P$.
Find the coordinates of $P$.

**[ 5 marks ]**

**Question 6**

Show that the derivative of the inverse cotangent function

$$y = \operatorname{arccot} x$$

is

$$\frac{dy}{dx} = -\frac{1}{1 + x^2}$$

The following trigonometry identity will be useful;

$$\cot^2 y + 1 = \csc^2 y$$

**[ 6 marks ]**

**Question 7**

The curve

$$y = \frac{2x + 1}{2x - 1}$$

crosses the $x$-axis at $A$ and the $y$-axis at $B$.

Find the point of intersection of the tangents to the curve at $A$ and $B$.

**[ 8 marks ]**