

## Lesson 14

## A-Level Pure Mathematics : Year 2 Differentiation III

### 14.1 Extension Material

Marks Available : 40

#### Question 1

Differentiate each of the following with respect to  $x$ ,

(i)  $y = e^x$       (ii)  $y = x^e$       (iii)  $y = e^e$

[ 6 marks ]

#### Question 2

Write down the derivative of each of the following with respect to  $x$ ,

(i)  $y = \ln x$       (ii)  $y = \ln x^2$       (iii)  $y = \ln\left(\frac{1}{x}\right)$

[ 6 marks ]

#### Question 3

Find the equation of the tangent to the curve

$$y = e^{\frac{1}{2}x}$$

at the point where it intercepts the  $y$ -axis.

Write your answer in the form  $ay + bx + c = 0$ , where  $a, b, c \in \mathbb{Z}$

[ 6 marks ]

**Question 4**

Find the points on the following curve where the gradient is 3

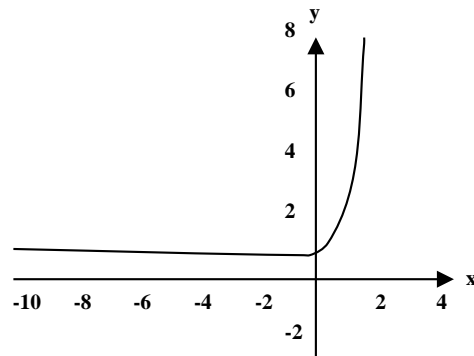
$$y = 5\sqrt{x} - \frac{1}{2}\ln x \quad x \in \mathbb{R}, x > 0$$

Give your answers correct to three significant figures.

[ 10 marks ]

### Question 5

A sketch graph of the function,  $f(x) = e^{2x} - e^x + 1$  is given below.



- (i) Find  $f'(x)$
- (ii) Explain, briefly, why the equation  $e^x = 0$  has no solution for  $x \in \mathbb{R}$
- (iii) Find the value of  $x$  such that  $f'(x) = 0$
- (iv) The graph of the  $f(x)$  has a turning point of the form  $(\ln a, b)$   
Determine the value of  $a$  and the value of  $b$
- (v) Find  $f''(x)$  at the turning point.
- (vi) Explain what your part (v) answer tells you about the turning point.

[ 12 marks ]

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