

Lesson 2

A-Level Pure Mathematics : Year 2 Differentiation III

2.1 The Product Rule

Given two functions, $u(x)$ and $v(x)$ that are multiplying each other, The Product Rule gives a method of obtaining the derivative of their product. It states that,

$$(u(x) v(x))' = u(x) v'(x) + u'(x) v(x)$$

All of the x in brackets are considered to be unnecessary clutter and so the rule is more usually written in the following succinct and elegant form,

The Product Rule

$$\text{If } f = uv \text{ then } f' = u v' + u' v$$

2.2 Example

Differentiate the product of x^5 with x^3 by immediately applying The Product Rule.

Teaching Video : <http://www.NumberWonder.co.uk/v9028/2.mp4>



Watch the video and
then write out the
solution here



[2 marks]

2.3 Exercise

Marks Available : 34

Question 1

- (i) Show how to use The Product Rule to differentiate,

$$y = x^9 \times x^{11}$$

[2 marks]

- (ii) Use algebra to simplify first, then differentiate,

$$y = x^9 \times x^{11}$$

[2 marks]

Question 2

- (i) Show how to use The Product Rule to differentiate,

$$y = 3x^5 \times 4x^7$$

[2 marks]

- (ii) Use algebra to simplify first, then differentiate,

$$y = 3x^5 \times 4x^7$$

[2 marks]

Question 3

- (i) Show how to use The Product Rule to differentiate,

$$y = x \times x$$

[2 marks]

- (ii) Use algebra to simplify first, then differentiate,

$$y = x \times x$$

[2 marks]

Question 4

- (i) Show how to use The Product Rule to differentiate,

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

[3 marks]

- (ii) Use algebra to simplify first, then differentiate,

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

[3 marks]

Question 5

- (i) Show how to use The Product Rule to differentiate,

$$y = x^{-3} \times x^8$$

[2 marks]

- (ii) Use algebra to simplify first, then differentiate,

$$y = x^{-3} \times x^8$$

[2 marks]

Question 6

- (i) Show how to use The Product Rule to differentiate,

$$y = (x^2 - 1)(x^2 + 1)$$

[3 marks]

- (ii) Use algebra to simplify first, then differentiate,

$$y = (x^2 - 1)(x^2 + 1)$$

[3 marks]

Question 7

Use the product rule to show that;

$$y = x^4 (3x^2 + 1)$$

has a first derivative given by,

$$\frac{dy}{dx} = 2x^3(9x^2 + 2)$$

and a second derivative given by,

$$\frac{d^2y}{dx^2} = 6x^2(15x^2 + 2)$$

before determining the third derivative.

[6 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk