Lesson 2

2.1 The Product Rule

Given two functions, $u(x)$ and $v(x)$ that are multiplying each other, The Product Rule gives a method of obtaining the derivative of their product. It states that,

$$\big(\, u(x)\; v(x)\,\big)' \;=\; u(x)\; v'(x) \;+\; u'(x)\; v(x)$$

All of the x in brackets are considered to be unnecessary clutter and so the rule is more usually written in the following succinct and elegant form,

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The Product Rule

$$\text{If } f \;=\; uv \quad \text{then} \quad f' \;=\; u\,v' \;+\; u'\,v$$

2.2 Example

Differentiate the product of x^5 with x^3 by immediately applying The Product Rule.

Teaching Video : http://www.NumberWonder.co.uk/v9028/2.mp4

Watch the video and
then write out the
solution here

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[2 marks]

2.3 Exercise

Marks Available : 34

Question 1

(i) Show how to use The Product Rule to differentiate,

$$y = x^9 \times x^{11}$$

[2 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = x^9 \times x^{11}$$

[2 marks]

Question 2

(i) Show how to use The Product Rule to differentiate,

$$y = 3x^5 \times 4x^7$$

[2 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = 3x^5 \times 4x^7$$

[2 marks]

Question 3

(i) Show how to use The Product Rule to differentiate,

$$y = x \times x$$

[2 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = x \times x$$

[2 marks]

Question 4

(i) Show how to use The Product Rule to differentiate,

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

[3 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

[3 marks]

Question 5

(i) Show how to use The Product Rule to differentiate,

$$y = x^{-3} \times x^{8}$$

[2 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = x^{-3} \times x^{8}$$

[2 marks]

Question 6

(i) Show how to use The Product Rule to differentiate,

$$y = \left(x^{2} - 1\right)\left(x^{2} + 1\right)$$

[3 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = \left(x^{2} - 1\right)\left(x^{2} + 1\right)$$

[3 marks]

Question 7

Use the product rule to show that;

$$y = x^4 \left(3x^2 + 1 \right)$$

has a first derivative given by,

$$\frac{dy}{dx} = 2x^3 \left(9x^2 + 2 \right)$$

and a second derivative given by,

$$\frac{d^2y}{dx^2} = 6x^2 \left(15x^2 + 2 \right)$$

before determining the third derivative.

[6 marks]