Lesson 3

A-Level Pure Mathematics : Year 2

Differentiation III

3.1 Product In, Product Out (PIPO)

When using The Product Rule, the object before differentiation is a product. It's considered elegant to have an object after the differentiation that's also a product. Furthermore, in tackling optimisation problems in which local minima and maxima are sought, (which correspond to where the derivative, the gradient, is zero) having a product equalling zero (rather than a sum) is a desirable situation.
In short, initially applying The Product Rule is often only half of a question; manipulating the algebra to derive an answer in the form of a product is the other.

3.2 Example

Show that the derivative of $y = x^3(2x + 5)^3$ can be expressed as,

$$\frac{dy}{dx} = 3x^2(2x + 5)^2(4x + 5)$$

Teaching Video : http://www.NumberWonder.co.uk/v9028/3.mp4

Watch the video and
then write out the
solution here

☞

[5 marks]

3.3 One For You To Do

Often, in the middle of using The Product Rule, The Chain Rule is required.

The Chain Rule for $y = [\, f(x) \,]^n$

$$\text{If } y = [\, f(x) \,]^n \text{ then } \frac{dy}{dx} = n\, [\, f(x) \,]^{n-1}\, f'(x)$$

The Product Rule

$$\text{If } f = uv \text{ then } f' = u\,v' + u'\,v$$

Try this problem, then check your solution with mine on the following page.

Try 1 Use the product rule to show that the derivative of,

$$y = (2x + 3)^2 (5x - 1)^3$$

is,

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2 (50x + 41)$$

[5 marks]

Answer to Try 1

$$y = (2x + 3)^2 \ (5x - 1)^3$$

$$\frac{dy}{dx} = (2x + 3)^2 \times 3(5x - 1)^2 \times 5 + 2\ (2x + 3)^1 \times 2 \times (5x - 1)^3$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2 \{ 15(2x + 3) + 4(5x - 1) \}$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2 \{ 30x + 45 + 20x - 4 \}$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2 \{ 50x + 41 \}$$

3.3 Exercise

Marks Available : 35

Question 1

Use the product rule to show that the derivative of,

$$y = x^5(x - 1)^2$$

is,

$$\frac{dy}{dx} = x^4(x - 1)(7x - 5)$$

[5 marks]

Question 2

Use the product rule to show that the derivative of,

$$y = x^7(6x + 5)^3$$

is,

$$\frac{dy}{dx} = 5x^6(6x + 5)^2(12x + 7)$$

[5 marks]

Question 3

Use the product rule to show that the derivative of,

$$y = (x^2 - 3)(x + 1)^2$$

is,

$$\frac{dy}{dx} = 2(x^2 - 1)(2x + 3)$$

[5 marks]

Question 4

Use the product rule to show that the derivative of,

$$y = \left(4x + 1\right)^{\frac{3}{2}}\left(x^2 + 5\right)$$

is,

$$\frac{dy}{dx} = 2\sqrt{4x + 1}\left(7x^2 + x + 15\right)$$

[5 marks]

Question 5

Use the product rule to show that the derivative of,

$$y = \left(2x - 3\right)^3\left(x^2 + 1\right)$$

is,

$$\frac{dy}{dx} = 2\left(2x - 3\right)^2\left(x^2 + 1\right)\left(7x^2 - 6x + 3\right)$$

[5 marks]

Question 6

Use the product rule to find the derivative of,

$$y = x^3(5x + 1)^2$$

Write your answer as a product.

[5 marks]

Question 7

Find the x component of the coordinates of the stationary points on the curve

$$y = \left(x^2 - 1\right)\sqrt{1 + x}$$

[5 marks]