Lesson 5

5.1 The Quotient Rule

Given two functions, $u(x)$ and $v(x)$, the first divided by the second, The Quotient Rule gives a method of obtaining the derivative of the division. It states that,

$$\left(\frac{u(x)}{v(x)}\right)' = \frac{v(x)\,u'(x) - v'(x)\,u(x)}{(v(x))^2}$$

All of the x in brackets are considered to be unnecessary clutter and so the rule is more usually written in the following succinct and elegant form,

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The Quotient Rule

$$\text{If } f = \frac{u}{v} \text{ then } f' = \frac{v\,u' - v'\,u}{v^2}$$

5.2 Example

Differentiate $y = \dfrac{x+4}{x+5}$ by immediately applying The Quotient Rule

Teaching Video : http://www.NumberWonder.co.uk/v9028/5.mp4

Watch the video and
then write out the
solution here

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[3 marks]

5.3 Exercise

Question 1

Given that $y = \dfrac{4x}{x + 3}$

use The Quotient Rule to show that the derivative is given by $\dfrac{dy}{dx} = \dfrac{12}{(x + 3)^2}$

[3 marks]

Question 2

Given that $y = \dfrac{x^2}{(x + 5)}$ use The Quotient Rule to show that

the derivative is given by $\dfrac{dy}{dx} = \dfrac{x(x + 10)}{(x + 5)^2}$

[3 marks]

Question 3

Given that $y = \dfrac{5x - 2}{3x + 1}$ use The Quotient Rule to show that

the derivative is given by $\dfrac{dy}{dx} = \dfrac{11}{(3x + 1)^2}$

[3 marks]

Question 4

Given that $y = \dfrac{x^2 + 1}{x^2 + 4}$ use The Quotient Rule to show that

the derivative is given by $\dfrac{dy}{dx} = \dfrac{6x}{(x^2 + 4)^2}$

[3 marks]

Question 5

Given that $y = \dfrac{x^5}{(2x + 1)^3}$ use The Quotient Rule to show that

the derivative is given by $\dfrac{dy}{dx} = \dfrac{x^4(4x + 5)}{(2x + 1)^4}$

[4 marks]

Question 6

Given that $y = \dfrac{x^7}{(3x + 2)^5}$ use The Quotient Rule to show that

the derivative is given by $\dfrac{dy}{dx} = \dfrac{2x^6(3x + 7)}{(3x + 2)^6}$

[5 marks]

Question 7

Given that $y = \dfrac{2(x+3)^3}{\sqrt{x}}$ use The Quotient Rule to

show that $\dfrac{dy}{dx} = \dfrac{(x+3)^2(5x-3)}{x^{\frac{3}{2}}}$

[4 marks]

Question 8

Given that $y = x^2\sqrt{x+5}$

use The Product Rule to show that $\dfrac{dy}{dx} = \dfrac{5x(x+4)}{2\sqrt{x+5}}$

[5 marks]

Question 9

$$f(x) = \frac{2x}{x + 5} + \frac{6x}{x^2 + 7x + 10} \qquad x > 0$$

(a)   Show that

$$f(x) = \frac{2x}{x + 2}$$

(b)   Hence find $f'(3)$

[5 marks]

Question 10

Given that the function $f(x) = \dfrac{x}{x^2 + 2}$ is increasing on the interval $[-k,\ k]$ find the largest possible value of k.

[**5 marks**]