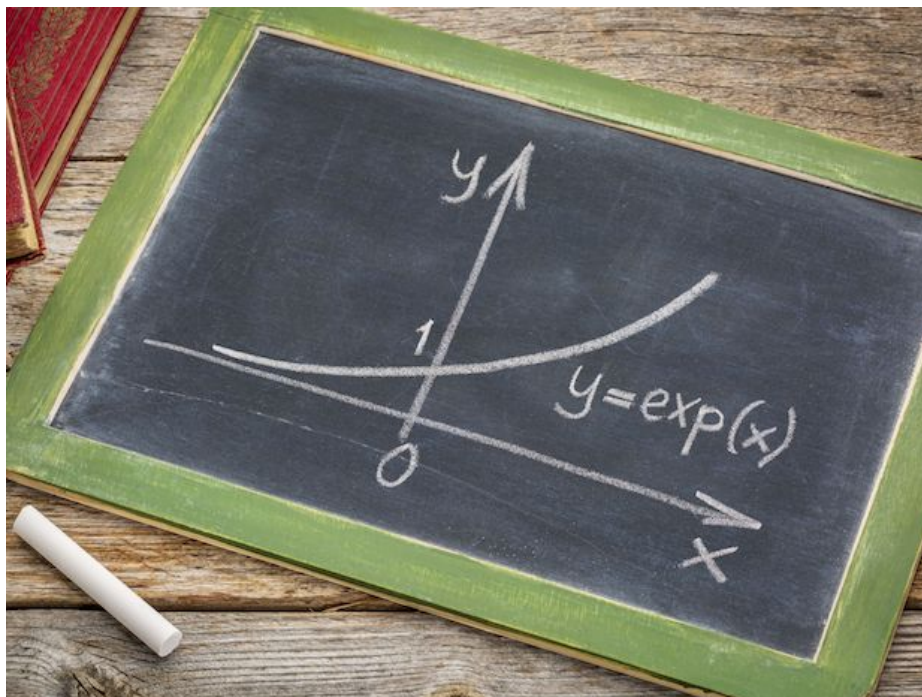


### 6.1 The Exponential Function



The function  $y = e^x$  has the remarkable property of being its own derivative.

---

#### The Derivative of $y = e^x$

$$\text{If } y = e^x \text{ then } \frac{dy}{dx} = e^x$$

---

### 6.2 The Value of e

$e$  is the number 2.71828 18284 59045 23536 02875 ...

Like  $\pi$  this is an irrational number and like  $\pi$  it crops up in many surprising places throughout mathematics.

### 6.3 Differentiating Exponentiated Functions

The Product Rule and The Quotient Rule can be applied to situations where the exponential function is involved. So too can The Chain Rule, as follows,

---

#### The Chain Rule for $y = e^{f(x)}$

$$\text{If } y = e^{f(x)} \text{ then } \frac{dy}{dx} = e^{f(x)} f'(x)$$

---

## 6.4 Examples

Differentiate each of the following,

( i )  $y = e^{8x^3+5x^2}$  ( Chain Rule Example )

( ii )  $y = x e^{3x}$  ( Product Rule Example )

( iii )  $y = \frac{x}{e^x}$  ( Quotient Rule Example )

Teaching Video : <http://www.NumberWonder.co.uk/v9028/6.mp4>



Watch the video and  
then write out the  
solutions here



[ 3, 3, 3 marks ]

## 6.5 Exercise

Marks Available : 50

### Question 1

Differentiate each of the following with respect to  $x$ ;

(i)  $y = e^{5x}$

(ii)  $y = e^{3x^5}$

[ 2, 2 marks ]

(iii)  $y = 8e^{6x} + 6e^{5x}$

(iv)  $y = 14e^x$

[ 2, 2 marks ]

(v)  $y = e^{-7x}$

(vi)  $y = \frac{4}{e^{5x}}$

[ 2, 2 marks ]

### Question 2

Remembering that when dividing same base indices subtract, differentiate the following with respect to  $x$ ,

$$y = \frac{e^{7x}}{e^{3x}}$$

[ 2 marks ]

### Question 3

By first expanding the brackets, differentiate with respect to  $x$ ,

$$y = e^{3x}(e^{4x} + 7e^{-9x})$$

[ 3 marks ]

**Question 4**

Use The Product Rule to differentiate each of the following with respect to  $x$ .  
Simplify to obtain an elegant answers.

(i)  $y = x^3 e^x$

(ii)  $y = x e^{-x}$

[ 3, 3 marks ]

**Question 5**

Use The Quotient Rule to differentiate each of the following with respect to  $x$ .  
Simplify to obtain elegant answers.

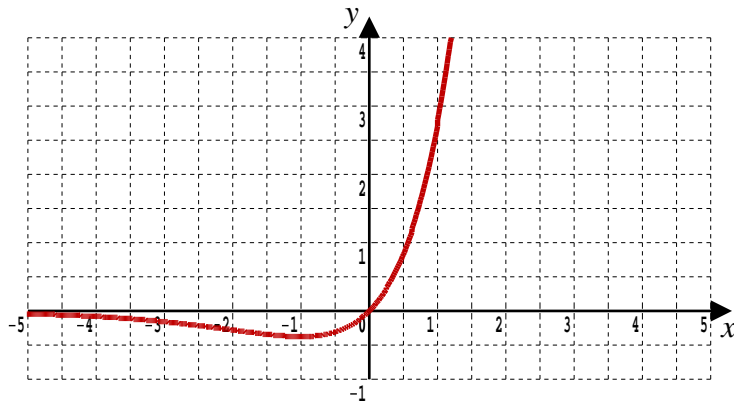
(i)  $y = \frac{e^x}{x}$

[ 3 marks ]

(ii)  $y = \frac{2 - x^2}{e^{2x}}$

[ 3 marks ]

**Question 6**



A curve has equation  $y = x e^x$

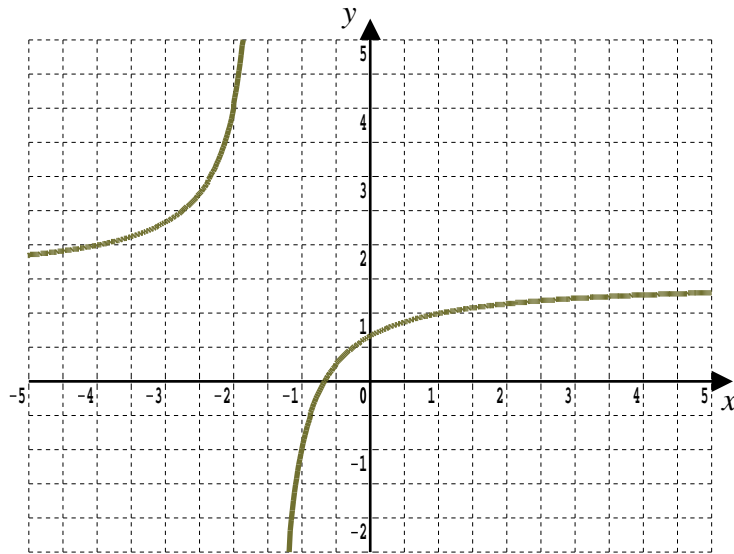
(i) Find  $\frac{dy}{dx}$

[ 3 marks ]

(ii) Find the equation of the tangent to the curve when  $x = 0$   
Draw this tangent onto the graph, above.

[ 4 marks ]

**Question 7**



A curve has equation,  $y = \frac{3x + 2}{2x + 3}$

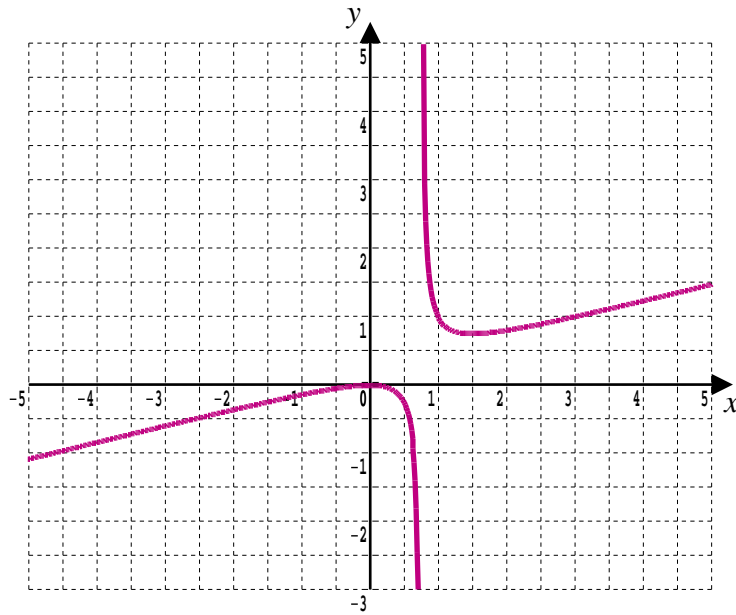
(i) Find  $\frac{dy}{dx}$

[ 3 marks ]

(ii) Find the equation of the normal to the curve when  $x = 1$   
Draw this normal onto the graph, above.

[ 4 marks ]

### Question 8



A curve has equation  $y = \frac{x^2}{(4x - 3)}$

- (i) Find an expression for  $\frac{dy}{dx}$

[ 3 marks ]

- (ii) Find the equation of the tangent to the curve when  $x = 1$   
Draw this tangent onto the graph, above.

[ 4 marks ]

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In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)