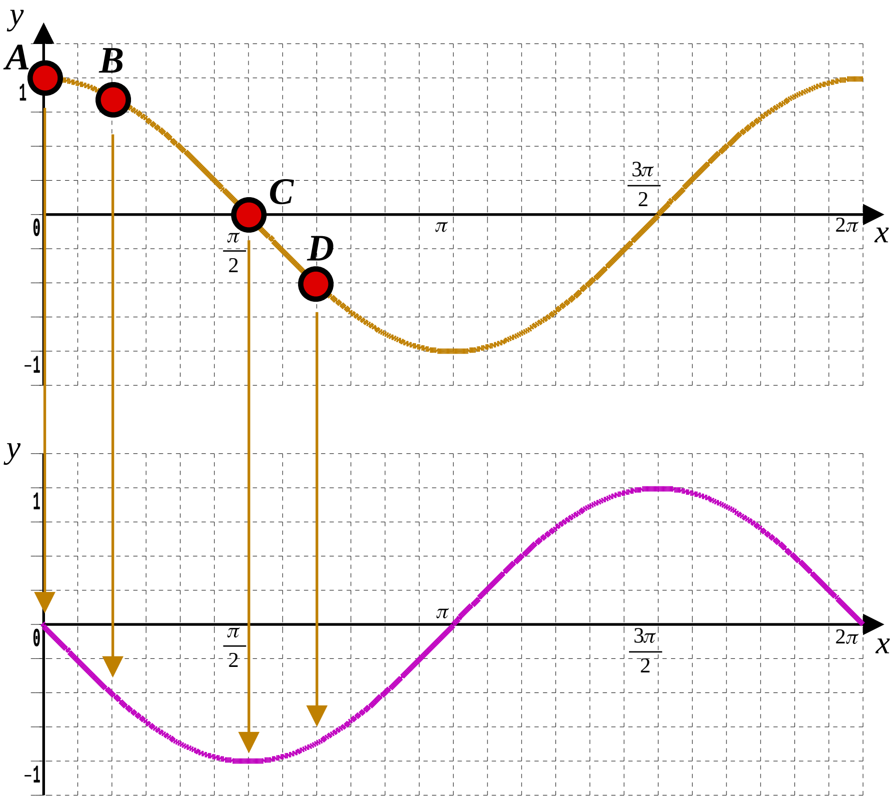
9.1 The Cosine Function

The upper graph shows the curve $y = \cos x$ with x measured in radians.
Of interest is the gradient of this graph.



Consider what the gradient of the upper graph is doing at the four points labelled.
A : At this maximum turning point the gradient is zero.
B : The gradient between A and B is negative and increasing in magnitude.
C : The curve seems to be sloping like the line $y = -x$, gradient -1
D : The gradient between C and D is negative and decreasing in magnitude.

Continuing in this manner, and plotting the gradients as a separate graph, the lower graph is obtained. It looks very much like the graph of $-\sin x$

This argument, although not a proof, is an intuitive reasoning of the important result that, provided radians are used,

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Derivative of Cosine

If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$ where x is in radians

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9.2 Differentiating Cosine Functions

The Product Rule and The Quotient Rule can be applied to situations where the cosine function is involved. So too can The Chain Rule, as follows,

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The Chain Rule for $y = cos\,(\,f\,(x))$

$$\text{If } y = cos\,(\,f\,(x)\,) \text{ then } \frac{dy}{dx} = -\,sin\,(\,f\,(x)\,) \times f\,'\,(x)$$

9.3 Example

Prove that a consequence of the derivative of $sin\,x$ being $cos\,x$ is that the derivative of $cos\,x$ is $(-\,sin\,x)$

Teaching Video : http://www.NumberWonder.co.uk/v9028/9.mp4

Watch the video and
then write out the
proof here

☞

[6 marks]

9.4 Exercise

Marks Available : 40

Question 1

Differentiate each of the following with respect to *x*,

(i) $y = cos\left(3x^5 + 2e^{3x} \right)$ **(ii)** $y = 5x^7 + cos^4 (3x)$

[2, 2 marks]

(iii) $y = cos\left(\frac{5}{x} + ln\, x \right), \quad x > 0$ **(iv)** $y = sin\left(1 - cos\, x \right)$

[2, 2 marks]

Question 2

By writing $y = sec\, x$ as $y = (cos\, x)^{-1}$ and using The Chain Rule, show that,

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Derivative of *sec x*

If $y = sec\, x$ then $\frac{dy}{dx} = sec\, x\, tan\, x$ where *x* is in radians

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[3 marks]

Question 3

By writing $y = tan\,x$ as $y = \dfrac{sin\,x}{cos\,x}$ and using The Quotient Rule, show that,

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Derivative of *tan x*

$$\text{If } y = tan\,x \text{ then } \frac{dy}{dx} = sec^2\,x \quad \text{where } x \text{ is in radians}$$

[3 marks]

Question 4

By writing $y = cot\,x$ as $y = \dfrac{cos\,x}{sin\,x}$ and using The Quotient Rule, show that,

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Derivative of *cot x*

$$\text{If } y = cot\,x \text{ then } \frac{dy}{dx} = -\,csc^2\,x \quad \text{where } x \text{ is in radians}$$

[3 marks]

Question 5

Table of Derivatives of Trigonometric Functions

$f(x)$	$f'(x)$	Told in Exam ?
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\tan x$	$\sec^2 x$	Yes
$\cot x$	$-\csc^2 x$	Yes

Use the above table of standard derivatives to differentiate each of the following with respect to x,

(i) $y = \tan(5x)$

(ii) $y = \sec\left(3x^4 + x^2\right)$

[2, 2 marks]

(iii) $y = \cot\left(5 - \ln x\right)$

(iv) $y = e^{\tan 4x}$

[2, 2 marks]

(v) $y = \ln(\sec x + \tan x)$

Simplify your answer as much as possible

[2 marks]

Question 6

From the small angle approximations it is known that,

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$
- $\tan\theta \approx \theta$

This allows a couple of useful deductions to be made, firstly that,

$$\lim_{h\to 0} \frac{\sin h}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

and secondly, that,

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = \lim_{h\to 0} \frac{1 - \frac{1}{2}h^2 - 1}{h} = \lim_{h\to 0} \left(-\frac{1}{2}h\right) \to 0$$

(i) Given that $f(x) = \cos x$, show from first principles that,

$$f'(x) = \lim_{h\to 0} \left(\left(\frac{\cos h - 1}{h} \right) \cos x - \frac{\sin h}{h} \sin x \right)$$

[4 marks]

(ii) Hence prove that $f'(x) = -\sin x$

[2 marks]

Question 7

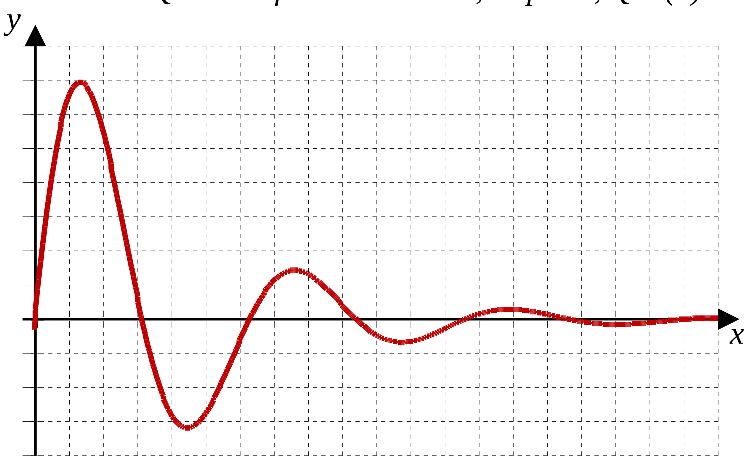
Given that $f(x) = csc\, x$ find the exact value of $f'\left(\frac{\pi}{6}\right)$

Simplify your answer.

[3 marks]

Question 8

A-Level Examination Question from June 2019, Paper 1, Q12(a)



$$f(x) = 10\, e^{-0.25x} \sin x \qquad\qquad x \geqslant 0$$

Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

[4 marks]