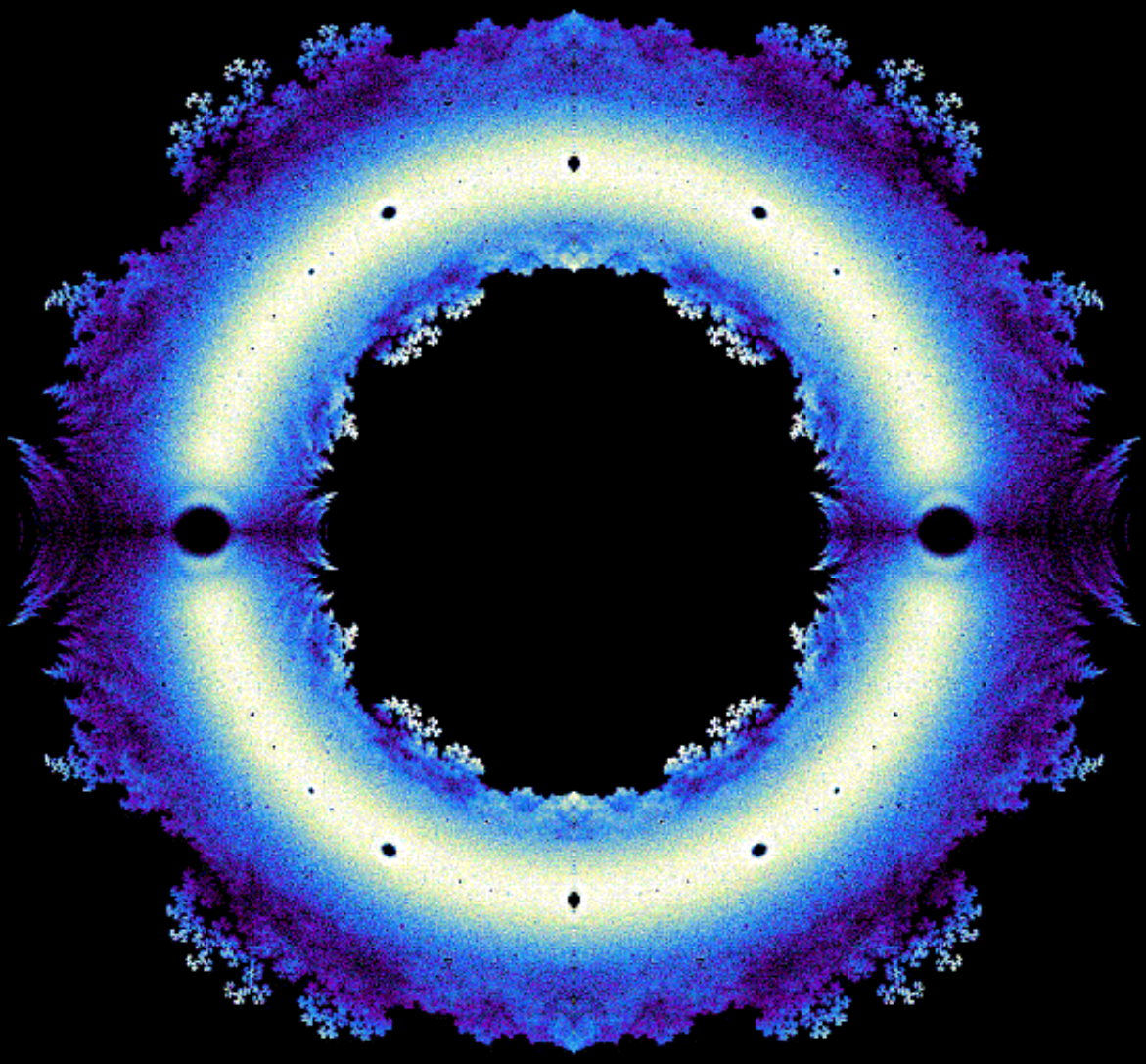
The Algebra Of Polynomials

The Factor and Remainder Theorems



Roots of all degree 18 polynomials with
coefficients +1 or –1 plotted in the complex plane
with roots on the real-axis removed
Image plotted Paul Nylander

The Algebra Of Polynomials

The Factor & The Remainder Theorems

Lesson 1

A-Level Pure Mathematics, Year 1
Additional Mathematics
The Algebra of Polynomials

1.1 What Are Polynomials ?

In this course, polynomials are algebraic expressions involving powers of x as the only function of x. The powers are integers that are non-negative. A typical polynomial is built by adding together multiples of the powers of x.

1.2 Exercise

Complete the following table,

Expression	Is the expression a polynomial ?	If the expression is not a polynomial, why not ?
$x^3 - x$		
$x^3 + 4.2x + 6.5$		
$x^2 + \sqrt{x}$		
$(x + 4)(x - 5)$		
$x^4 + \sqrt{3}\,x^2 + \pi^2$		
$x^{10} + x^1 + x^{0.1}$		
$x + \frac{1}{x}$		
$\sqrt{5}\,x^3 + x^{\sqrt{2}} - 41x + 20$		
$2\sin^2 x + 3\sin x + 12$		

[18 marks]

Polynomials are amongst the simplest form of mathematical function. In spite of this, they are remarkably useful. They are easy to integrate and differentiate and many other, more complicated functions, can be approximated by polynomials. They have a structure in the world of algebra that is similar to that of the integers in the world of numbers.

1.3 The Degree Of A Polynomial

The ***degree*** of a polynomial is the index of the highest power of x in the expression. A quadratic equation is an example of a polynomial of degree two. Polynomials of higher degree include;

Cubics (degree 3), Quartics (degree 4), Quintics (degree 5), Sextics (degree 6)

and....

... especially useful to medical students...

Septics (degree 7)

1.4 Exercise

Complete the following table,

Polynomial Expression	What is the degree and name of the polynomial ?
$x^5 + 4x^3 + 576x^2 + 1$	
$(2x + 7)(5x - 8)$	
$62 + 13x^4 - x^6$	
$(4 + x)(3x + 4)(1 - x)$	
$(x^2 + 1)(x^2 - 1)$	
$x^3(3x - 4 + x^4)$	
$3x - 5$	

[7 marks]