Lesson 6

6.1 Revision

The Factor Theorem

If, for a given polynomial function $p(x)$, $p(a) = 0$ (for some constant, a)

then $(x - a)$ is a factor of $p(x)$

The Remainder Theorem

When a polynomial $p(x)$ is divided by $(x - a)$, where a is a constant,

the remainder is $p(a)$

6.2 The Revision

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 50

Question 1

$$p(x) = 2x^3 + 4x^2 - 6x + 14$$

What is the degree of polynomial $p(x)$?

[**1 mark**]

Question 2

A polynomial of degree five is called a Quintic.

(**i**)   What is a polynomial of degree 2 called ?

[**1 mark**]

(**ii**)   What is a polynomial of degree 4 called ?

[**1 mark**]

Question 3

A-Level Examination Question from May 2011, Paper C2, Q1 (Edexcel)

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a)   Find the remainder when $f(x)$ is divided by $(x - 1)$

[2 marks]

(b)   Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$

[2 marks]

(c)   Factorise $f(x)$ completely

[4 marks]

Question 4

A-Level Examination Question from January 2011, Paper C2, Q1 (Edexcel)

$$f(x) = x^4 + x^3 + 2x^2 + ax + b \quad \text{where } a \text{ and } b \text{ are constants}$$

When $f(x)$ is divided by $(x - 1)$ the remainder is 7

(a)　Show that $a + b = 3$

[2 marks]

When $f(x)$ is divided by $(x + 2)$ the remainder is -8

(b)　Find the value of a and the value of b

[5 marks]

Question 5

A-Level Examination Question from January 2008, Paper C2, Q1 (Edexcel)

(**a**) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(**i**) $x - 3$

(**ii**) $x + 2$

[3 marks]

(**b**) Hence, or otherwise, find all solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0$$

[4 marks]

Question 6

Additional Mathematics Examination Question from June 2012, Q3 (OCR)

The function $f(x) = x^3 + ax + 6$ is such that when $f(x)$ is divided by $(x - 3)$ the remainder is 12

(i)   Show that the value of a is -7

[2 marks]

(ii)   Factorise $f(x)$

[3 marks]

Question 7

A-Level Examination Question from January 2012, Paper C2, Q5 (Edexcel)

$$f(x) = x^3 + a x^2 + bx + 3 \text{ where } a \text{ and } b \text{ are constants}$$

Given that when $f(x)$ is divided by $(x + 2)$ the remainder is 7,

(a) Show that $2a - b = 6$

[2 marks]

Given also that when $f(x)$ is divided by $(x - 1)$ the remainder is 4

(b) Find the value of a and the value of b

[4 marks]

Question 8

A-Level Examination question from June 2008, Paper C2, Q1 (Edexcel)

$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

(a)   Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$

[2 marks]

(b)   Factorise $f(x)$ completely

[4 marks]

Question 9

A-Level Examination Question from January 2005, Paper C2, Q5 (Edexcel)

$$f(x) = x^3 - 2x^2 + ax + b \text{ where } a \text{ and } b \text{ are constants}$$

- When $f(x)$ is divided by $(x - 2)$ the remainder is 1
- When $f(x)$ is divided the $(x + 1)$ the remainder is 28

(**a**)   Find the value of a and the value of b

[6 marks]

(**b**)   Show that $(x - 3)$ is a factor of $f(x)$

[2 marks]