Lesson 10

A-Level Pure Mathematics, Year 1
Additional Mathematics
Coordinate Geometry

10.1 Tangent from Curve

Previously looked at was "How to find the equation of the tangent to a circle". The mathematics needed to find the tangents to more general curves is straight forward, provided an ability to *differentiate* is already in place.

10.2 Differentiation

This is the remarkable ability to take the equation of many curves and, without any working, simply write down the gradient equation of that curve.

The Power Rule

$$\text{If } \quad y = x^n \quad \text{ then } \quad \frac{dy}{dx} = n\,x^{n-1} \quad \text{ for any constant, } n$$

Example #1

Write down the derivative of the following curve then check your answer with that given at the foot of the next page.

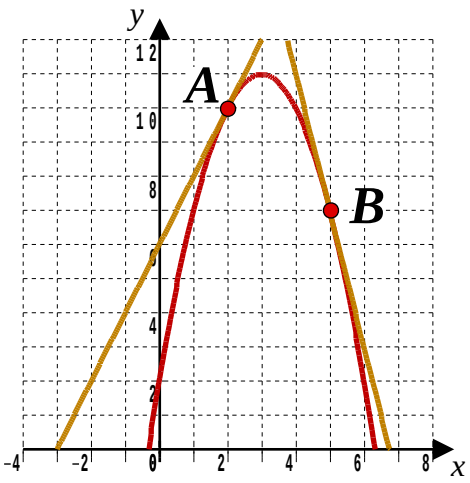
$$y = 7x^4 + 0.5x + 17$$

$$\Rightarrow \frac{dy}{dx} =$$

[2 marks]

10.3 The Key Idea

The gradient of the tangent to a point on a curve is the same as the gradient of the the curve at that point.



The red parabola, P, has equation $y = 2 + 6x - x^2$ and so $\frac{dy}{dx} = 6 - 2x$

At $A(2,\ 10)$, P has gradient 2 and the tangent is $y = 2x + 6$

At $B(5,\ 7)$, P has gradient -4 and the tangent is $y = -4x + 27$

10.4 Making Use of The Key Idea

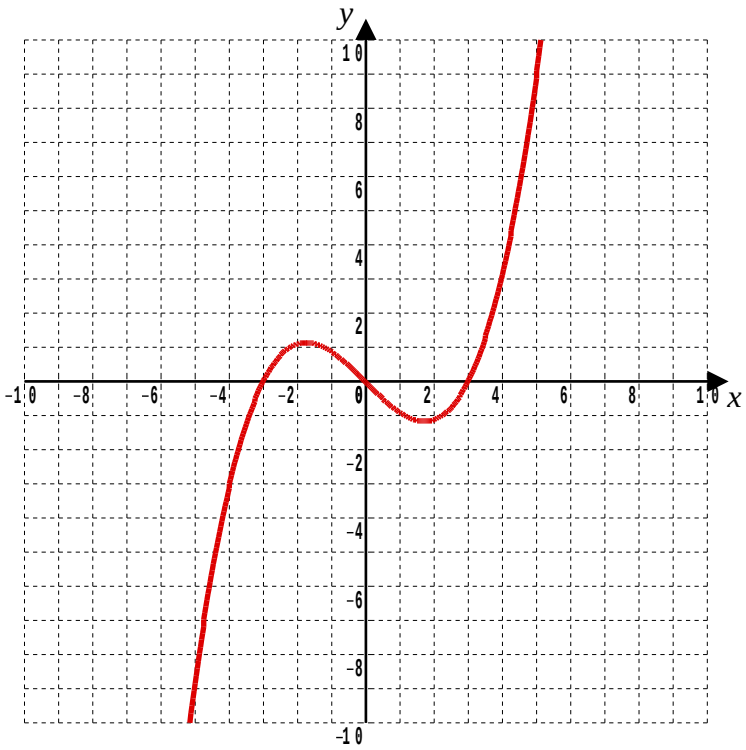
The key idea is often used to get the equation of the tangent to a curve at a given point starting from only the equation of the curve.

Example #2

The equation of a curve is $y = \frac{x^3}{9} - x$

(i) Find the equation of the tangent to this curve at the point (3, 0)

(ii) To the graph below add the part (i) tangent



Teaching Video : http://www.NumberWonder.co.uk/v9033/10.mp4

[6 marks]

10.5 Example #1 Answer : $y = 7x^4 + 0.5x + 17 \Rightarrow \frac{dy}{dx} = 28x^3 + 0.5$

10.6 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Differentiate

$$y = 4x^8 - 3x^5$$

[2 marks]

Question 2

Write down the derivative of

$$y = 15x^{0.4}$$

[2 marks]

Question 3

Differentiate

$$y = 7 + \frac{1}{2}x^6$$

[2 marks]

Question 4

Determine $\frac{dy}{dx}$ of the following expression,

$$y = 0.2x^9 + 0.1x$$

[2 marks]

Question 5

Write down the gradient equation for the following curve

$$y = x^{-5}$$

[2 marks]

Question 6

By first expanding the brackets, differentiate the following

$$y = (3x + 8)(2x + 1)$$

[3 marks]

Question 7

Find the derivative of the following

$$y = \frac{x^3}{9} + 4$$

[2 marks]

Question 8

(i) Differentiate;

$$y = \frac{12}{x^2}$$

[2 marks]

Question 9

Find the numerical value of the gradient at the point (1, 5) on the curve

$$y = x^3 + 4$$

[3 marks]

Question 10

Find the numerical value of the gradient at the point (2, 3.2) on the curve,

$$y = 0.1\,x^5$$

[2 marks]

Question 11

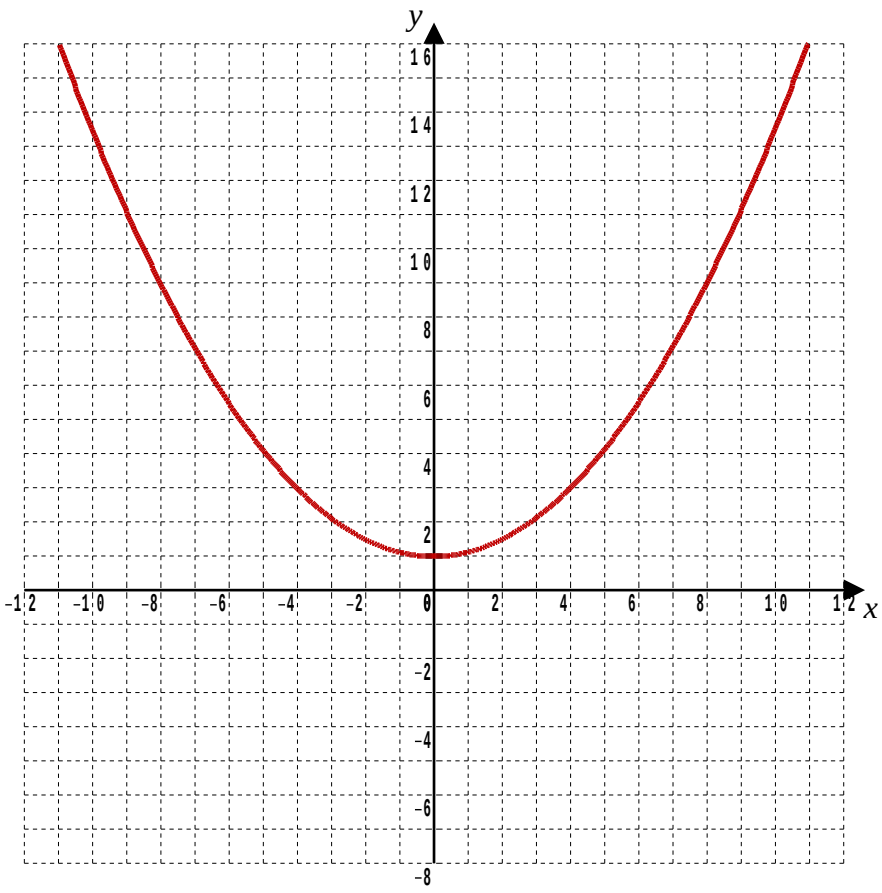
The equation of a curve is

$$y = \frac{x^2}{8} + 1$$

(i) Find the equation of the tangent to this curve at the point (8, 9)

[4 marks]

(ii) To the graph of the curve add your part (i) tangent



[2 marks]

Question 12

Additional Mathematics Examination Question from June 2011, Q2 (OCR)

The equation of a curve is

$$y = x^3 - x^2 - 2x - 3$$

Find the equation of the tangent to this curve at the point (3, 9)

[5 marks]

Question 13

Additional Mathematics Examination Question from June 2007, Q6 (OCR)

Find the equation of the tangent to the curve

$$y = x^3 - 3x + 4$$

at the point (2, 6)

[4 marks]

Question 14

$$f(x) = \frac{x^3 - x + 8}{16x}, \qquad x > 0$$

(i) Show that $f(x) = Ax^2 + B + Cx^{-1}$, where A, B and C are constants to be determined.

[4 marks]

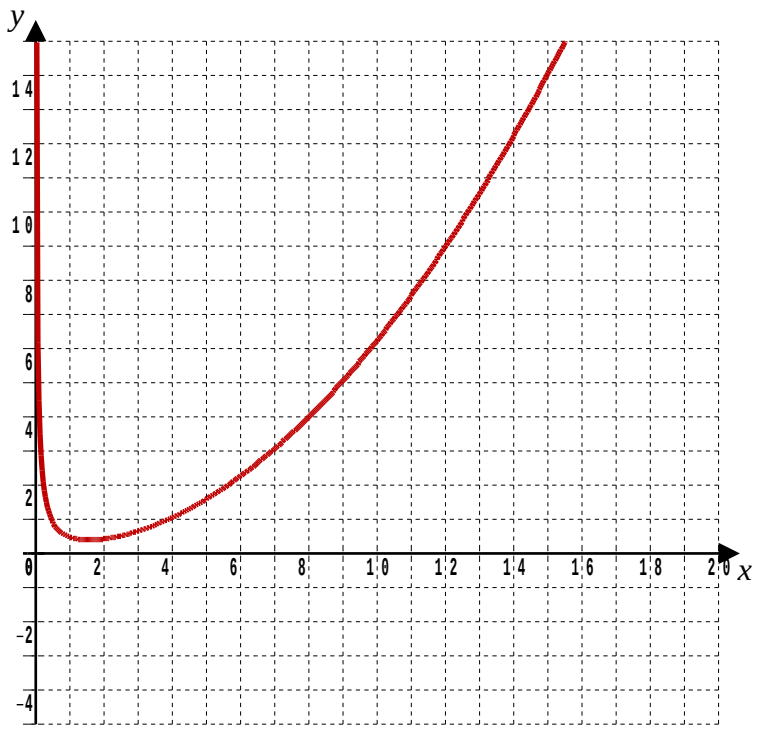
(ii) Hence find $f'(x)$

[2 marks]

(iii) Find an equation of the tangent to the curve $y = f(x)$ when $x = 8$

[4 marks]

(iv) The graph of $f(x)$ is plotted below.
Add the tangent calculated in part (iii) to this graph.



[3 marks]