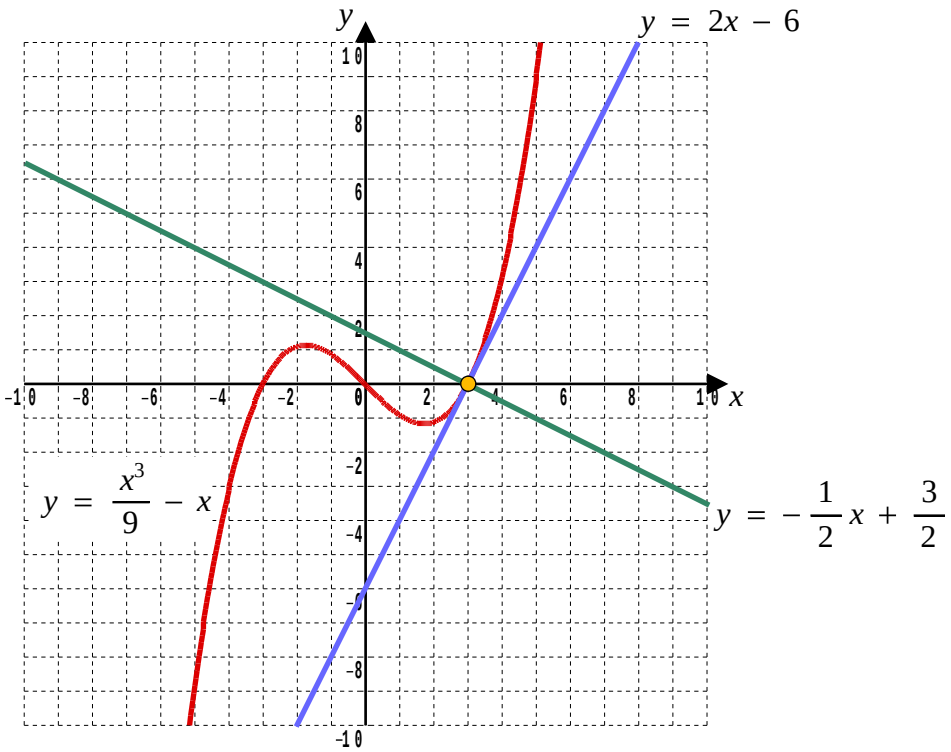
Lesson 11

11.1 Normal from Curve

Previously, the curve with equation $y = \frac{x^3}{9} - x$ was studied and the tangent to it at the point $(3, 0)$ found to be $y = 2x - 6$



There is a second line of interest, called the "normal" that is a right angles to the tangent at any specified point. At the point $(3, 0)$ the normal to the curve $y = \frac{x^3}{9} - x$ turns out to be $y = -\frac{1}{2}x + \frac{3}{2}$.

Notice that the gradient of the tangent, m_t, and the gradient of the normal, m_n have the property of any pair of mutually perpendicular lines; $m_t \times m_n = -1$

In other words, each is the sign changed reciprocal of the other.

11.2 Why the Normal is of Interest

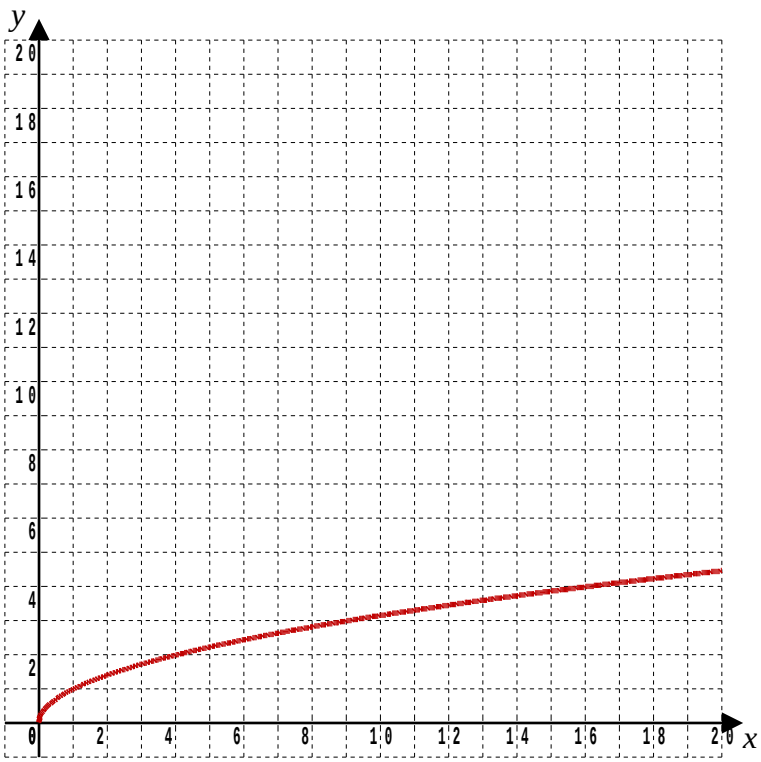
Imagine the graph to be a road map and the curve a road on that map. A car moves along the road with constant speed. The tangent represents the direction a car moving along the road has at any moment. The normal represents the direction along which the force felt by a person in the car acts as it moves around each bend. Like the tangent the normal gives only a direction. It does not give the magnitude of the force; that depends on how sharply the road is bending and, indeed, on a straight piece of road the force along the normal has magnitude zero. The force along the normal is often referred to as centripetal force.

11.3 Example

The equation of a curve is $y = \sqrt{x}$

(i) Find the equation of the normal to this curve at the point where $x = 4$

(ii) To the graph below add the part (i) normal.



Teaching Video : http://www.NumberWonder.co.uk/v9033/11.mp4

[5 marks]

11.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

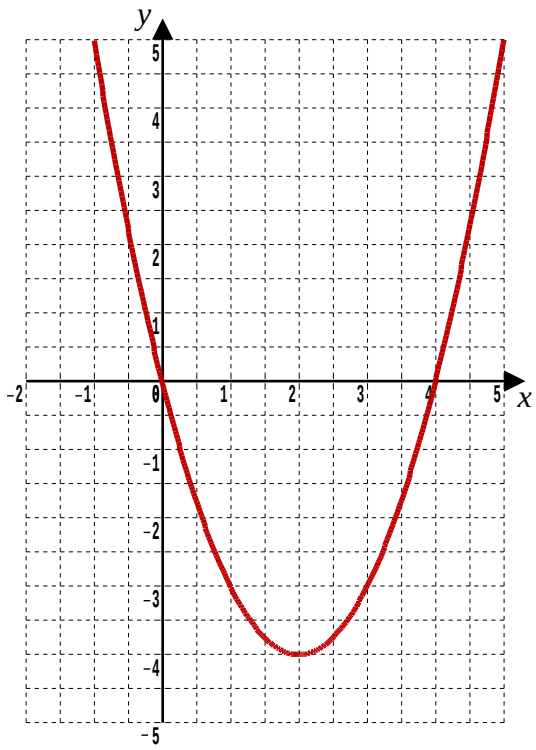
Marks Available : 52

Question 1

The equation of a curve is $y = x^2 - 4x$

(i) Find the equation of the normal to this curve at the point where $x = 4$

(ii) To the graph below add the part (i) normal



[5 marks]

Question 2

Additional Mathematics Examination Question from June 2009, Q2 (OCR)

Find the equation of the normal to the curve

$$y = x^3 + 5x - 7$$

at the point $(1, -1)$

[5 marks]

Question 3

Additional Mathematics Examination Question from June 2019, Paper 1, Q3 (OCR)

Find the equation of the normal to the curve

$$y = x^3 - 2x^2 + 2x + 4$$

at the point $(2, 8)$

[6 marks]

Question 4

Additional Mathematics Examination Question from June 2018, Q7 (OCR)

(i) Find the coordinates of the points where the line $y = 7x - 9$ cuts the curve $y = x^2 + 2x - 5$

[4 marks]

(ii) Determine whether the line is a normal to the curve at either of the points of intersection

[3 marks]

Question 5

Additional Mathematics Examination Question from June 2014, Q10 (OCR)

(i) Find the coordinates of the point P on the curve $y = 2x^2 + x - 5$ where the gradient of the curve is 5

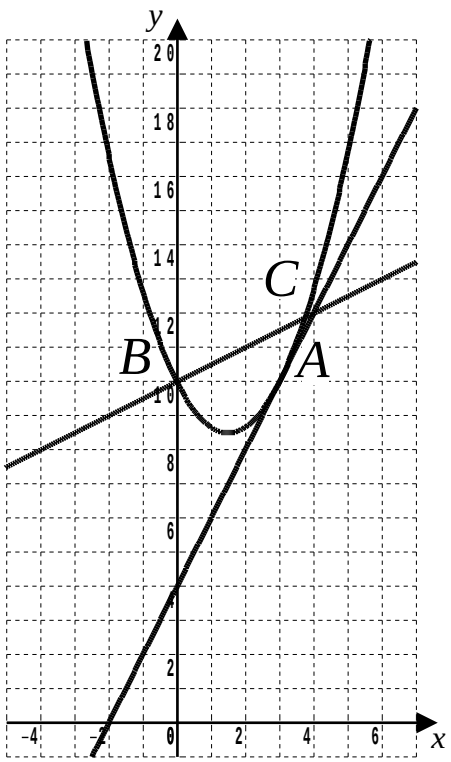
[3 marks]

(ii) Find the equation of the normal to the curve at the point P

[3 marks]

Question 6

Additional Mathematics Examination Question from June 2005, Q10 (OCR)



The curve shown has equation;

$$y = \frac{2}{3}x^2 - 2x + 10$$

(i) Find the equation of the tangent to the curve at A (3, 10)

[4 marks]

(ii) Show that the equation of the normal to the curve at B (0, 10) is

$$2y - x = 20$$

[3 marks]

(iii) Find the coordinates of the point C where these two lines intersect

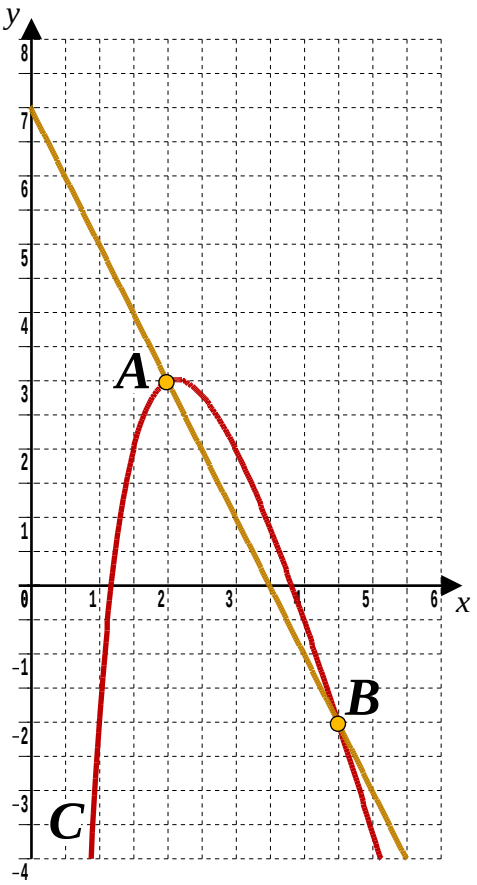
[3 marks]

(iv) Calculate the length BC

[2 marks]

Question 7

A-Level Examination Question from May 2014, IAL, Paper C1(R), Q11 (Edexcel)



The sketch is of part of the curve C with equation $y = 20 - 4x - \frac{18}{x}$, $x > 0$

Point A lies on C and has an x coordinate equal to 2

(a) Show that the equation of the normal to C at A is $y = -2x + 7$

[6 marks]

The normal to C at A meets C again at the point B

(b) Use algebra to find the coordinates of B

[5 marks]