**Lesson 4**

**A-Level Pure Mathematics, Year 1**
**Additional Mathematics**
**GCSE**
**Coordinate Geometry**

## 4.1 Lines at Right Angles

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**The Perpendicular Lines Theorem (Version 1)**

If the gradient of the line $l_1$ is $m_1$ and the gradient of the line $l_2$ is $m_2$ then the lines $l_1$ and $l_2$ are perpendicular if and only if

$$m_1 \times m_2 = -1$$

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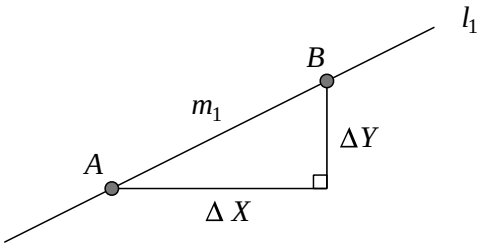
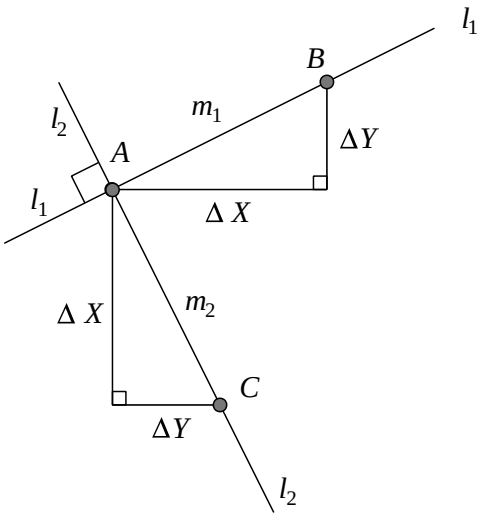
**Proof**

Consider two points, $A$ and $B$ on the line $l_1$ which has gradient $m_1$




Clearly, $m_1 = \dfrac{\Delta Y}{\Delta X}$

Now, consider a rotation of $-90°$ about the point $A$ which gives a line $l_2$ with gradient $m_2$ which is perpendicular to $l_1$.




Clearly, $m_2 = -\dfrac{\Delta X}{\Delta Y}$

Observe that, $m_1 \times m_2 = \dfrac{\Delta Y}{\Delta X} \times -\dfrac{\Delta X}{\Delta Y}$

$= -1$ □

In many questions, the gradient of a first line will be known and the gradient of a second, perpendicular to the first, sought.
In consequense the following version of the theorem is often of more use;

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**The Perpendicular Lines Theorem (Version 2)**

Given a line $l_1$ with gradient $m_1$ then the gradient $m_2$ of any perpendicular line $l_2$ is the ***sign changed reciprocal*** of $m_1$.

$$\text{That is, } m_2 = -\frac{1}{m_1}$$

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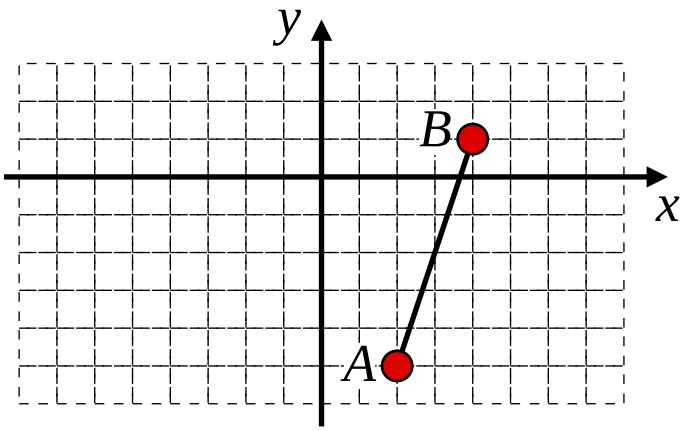
**4.2 Example**

Find the equation of the perpendicular bisector of the line segment $AB$ where $A$ is $(2, -5)$ and $B$ is $(4, 1)$

**( i )** Give your answer in the form $y = mx + c$

**( ii )** Illustrate your answer with a sketch graph.




Teaching Video : http://www.NumberWonder.co.uk/v9033/4.mp4

**[ 4 marks ]**

**4.3 Exercise**

*Any solution based entirely on graphical or numerical methods is not acceptable*

Marks Available : 50

**Question 1**

A line, $L$, has equation

$$y = \frac{2}{3}x + \frac{1}{3}$$

**( i )** What is the gradient of $L$ ?

**[ 1 mark ]**

**( ii )** What would be the gradient of a line, perpendicular to $L$ ?

**[ 1 mark ]**

**Question 2**

A line has equation $2x + 5y - 4 = 0$

**( i )** Write this line's equation in the form $y = mx + c$

**[ 1 marks ]**

**( ii )** Hence state the gradient of the line $2x + 5y - 4 = 0$

**[ 1 mark ]**

**( iii )** What is the gradient of a line, perpendicular to $2x + 5y - 4 = 0$ ?

**[ 1 mark ]**

**Question 3**

A line has equation $5x - 3y - 2 = 0$

**( i )** What is the gradient of this line ?

**[ 2 marks ]**

**( ii )** What is the gradient of a line perpendicular to $5x - 3y - 2 = 0$ ?

**[ 1 mark ]**

**Question 4**

*Additional Mathematics Examination Question from June 2015, Q1, (OCR, FSMQ)*

Find the equation of the line which is perpendicular to the line $2x + 3y = 5$ and which passes through the point $(3, 4)$

**[ 3 marks ]**

**Question 5**

*A-Level Examination question from May 2011, C1, Q3 (Edexcel)*

The points $P$ and $Q$ have coordinates $(-1, 6)$ and $(9, 0)$ respectively.
The line $l$ is perpendicular to $PQ$ and passes through the mid-point of $PQ$.
Find an equation for $l$, giving your answer in the form $ax + by + c = 0$,
where $a$, $b$ and $c$ are integers.

**[ 5 marks ]**

**Question 6**

*A-Level Examination Question from January 2010, C1, Q3 (Edexcel)*

The line $l_1$ has equation $3x + 5y - 2 = 0$

**( a )** Find the gradient of $l_1$

**[ 2 marks ]**

The line $l_2$ is perpendicular to $l_1$ and passes through the point ( 3, 1 )

**( b )** Find the equation of $l_2$ in the form $y = mx + c$, where $m$ and $c$ are constants

**[ 3 marks ]**

**Question 7**

*A-Level Examination Question from January 2006, C1, Q3 (Edexcel)*

The line $L$ has equation $y = 5 - 2x$

**( a )** Show that the point $P(3, -1)$ lies on $L$

**[ 1 mark ]**

**( b )** Find an equation of the line perpendicular to $L$, which passes through $P$. Give your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**[ 4 marks ]**

**Question 8**

*Additional Mathematics Examination Question from June 2014, Q8 (OCR)*

Four points have coordinates $A(-5, -1)$, $B(0, 4)$, $C(7, 3)$ and $D(2, -2)$

**( i )** Using gradients of lines, prove that *ABCD* is a parallelogram

**[ 2 marks ]**

**( ii )** Using lengths of lines, prove that *ABCD* is a rhombus

**[ 2 marks ]**

**( iii )** Prove that *ABCD* is not a square

**[ 2 marks ]**

**Question 9**

*A-Level Examination Question from January 2011, C1, Q9 (Edexcel)*

The line $L_1$ has equation $2y - 3x - k = 0$, where $k$ is a constant.

Given that the point $A(1, 4)$ lies on $L_1$ find,

**( a )** the value of $k$,

**[ 1 mark ]**

**( b )** the gradient of $L_1$

**[ 2 marks ]**

The line $L_2$ passes through $A$ and is perpendicular to $L_1$

**( c )** Find an equation of $L_2$ giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**[ 4 marks ]**

The line $L_2$ crosses the $x$-axis at the point $B$

**( d )** Find the coordinates of $B$

**[ 2 marks ]**

**( e )** Find the exact length of $AB$

**[ 2 marks ]**

**Question 10**

*A-Level Examination Question from May 2007, C1, Q11 (Edexcel)*

The line $l_1$ has equation $y = 3x + 2$,

and the line $l_2$ has equation $3x + 2y - 8 = 0$

**( a )** Find the gradient of the line $l_2$

**[ 2 marks ]**

The point of intersection of $l_1$ and $l_2$ is $P$

**( b )** Find the coordinates of $P$

**[ 3 marks ]**

The lines $l_1$ and $l_2$ cross the line $y = 1$ at the points $A$ and $B$ respectively.

**( c )** Find the area of triangle $ABP$

**[ 4 marks ]**