Lesson 6

A-Level Pure Mathematics, Year 1
Additional Mathematics
Coordinate Geometry

6.1 The Equation of a Circle

In the work of straight lines, a geometric object was analysed using algebra. Extensive use was made of the equation $y = mx + c$ and an understanding of how the various parts of that equation related to the particular straight line under consideration; that the value of m gave information about gradient, and that the value of c gave information about where the line intercepted the y-axis.

A similar approach is taken with circles.

The Equation of a Circle

$$(x - a)^2 + (y - b)^2 = r^2$$

represents a circle with centre (a, b) and radius r

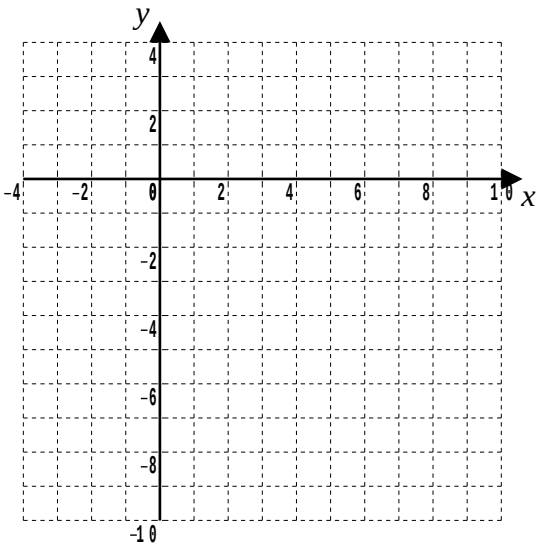
6.2 Example

For the circle with equation $(x - 4)^2 + (y + 3)^2 = 25$

(i) Write down the coordinates of the circle's centre and state its radius.

(ii) Prove that the circle passes through the origin.

(iii) Sketch the circle, using the NEWS method.



[2, 2, 2 marks]

Teaching Video : http://www.NumberWonder.co.uk/v9033/6.mp4

The video walks through the above example

6.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 60

Question 1

$$(x - 16)^2 + (y - 9)^2 = 100$$

State the radius and the centre of this circle

[2 marks]

Question 2

$$(x + 25)^2 + (y + 121)^2 = 17^2$$

State the radius and the centre of this circle

[2 marks]

Question 3

$$(x - 12)^2 + (y + 5)^2 = 400$$

(i) State the radius and the centre of this circle, *C*

[2 marks]

(ii) What is the distance between the centre of the circle, *C*, and the origin, *O* ?

[2 marks]

Question 4

Write down the equation of a circle, centre (2, 8) and radius 7

[2 marks]

Question 5

Write down the equation of a circle, centre (– 2.1, 4.8) and radius 3.6

[2 marks]

Question 6

Write down the equation of a circle, centre (– 1, – 5) and radius $\sqrt{13}$

[2 marks]

Question 7

Additional Mathematics Examination Question from June 2006, Q4 (OCR)

(i) Find the distance between the points (2, 3) and (7, 9)

[2 marks]

(ii) Hence find the equation of the circle with centre (2, 3) and passing through the point (7, 9)

[2 marks]

Question 8

Additional Mathematics Examination Question from June 2009, Q4, (OCR)

AB is a diameter of a circle, where *A* is (1, 1) and *B* is (5, 3)

Find,

(i) the exact length of *AB*

[2 marks]

(ii) the coordinates of the midpoint of *AB*

[1 mark]

(iii) the equation of the circle

[3 marks]

Question 9

Additional Mathematics Examination Question from June 2011, Q1 (OCR)

Determine whether the point (5, 2) lies inside or outside the circle whose equation is $x^2 + y^2 = 30$

You must show your working.

[3 marks]

Question 10

Consider the circle with equation $(x - 10)^2 + (y - 8)^2 = 100$

(i) Write down the coordinates of the circle's centre

[1 mark]

(ii) What is the radius of the circle ?

[1 mark]

(iii) Prove that the point (18, 14) is on the circle

[2 marks]

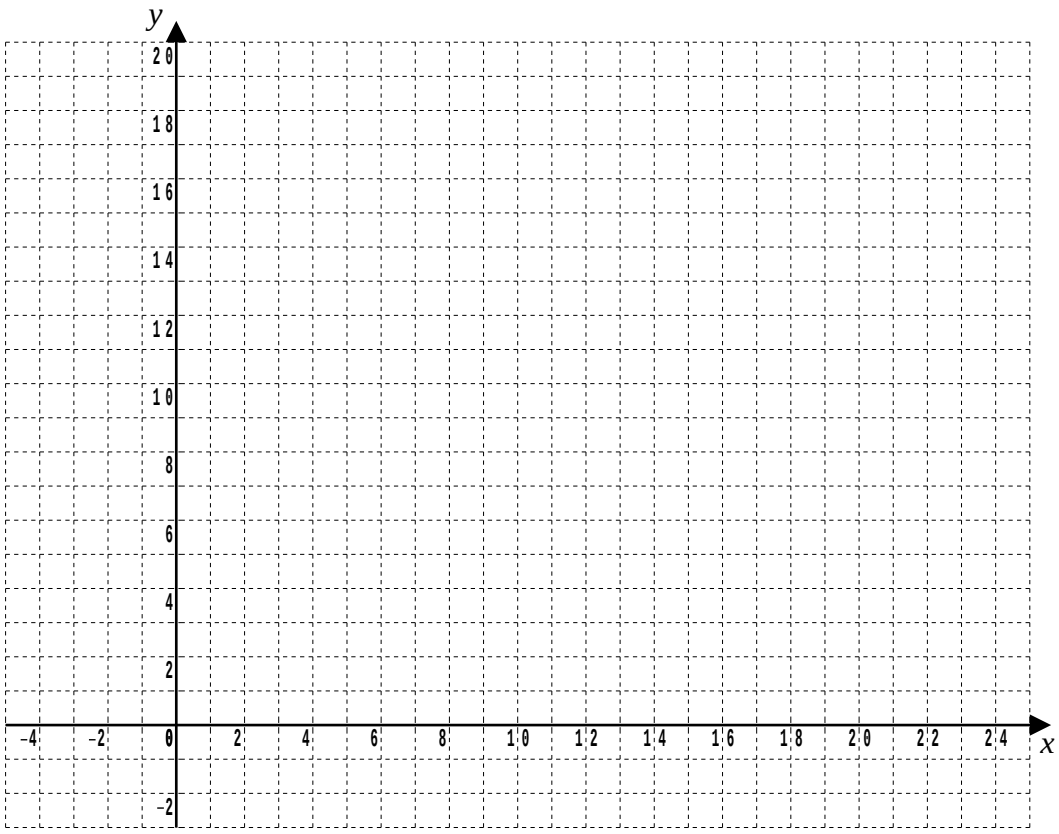
(iv) Find the coordinates of the two points where the circle crosses the *x*-axis

[2 marks]

(v) Sketch the circle by first plotting points

- with the help of the NEWS method
- using the answers to parts (iii) and (iv)

and making use of the circle's horizontal and vertical lines of mirror symmetry



[3 marks]

Question 11

Additional Mathematics Examination Question from May 2012, Q10 (OCR)

$A(1, 10)$, $B(8, 9)$ and $C(7, 2)$ are three points

(i) Find the coordinates of the midpoint, *M*, of *AC*

[1 mark]

(ii) Find the equation of the circle with *AC* as diameter

[4 marks]

(iii) Show that *B* lies on this circle

[1 mark]

(iv) Prove that *AM* and *BM* are perpendicular

[3 marks]

(v) *BD* is a diameter of this circle. Find the coordinates of *D*

[3 marks]

Question 12

Additional Mathematics Examination Question from June 2013, Q11 (OCR)

A circle has equation $(x - 2)^2 + y^2 = 100$

(a) Write down the radius and the coordinates of the centre, *C*, of this circle

[2 marks]

The line $y = 2x + 6$ cuts the circle at two points, *A* and *B*

(b) Find

(i) the coordinates of *A* and *B*

[5 marks]

(ii) the midpoint, *M*, of *AB*

[1 mark]

(iii) the length *AB*

[2 marks]

(c) Hence find the distance of the centre of the circle from the line *AB*

[2 marks]