Lesson 7

A-Level Pure Mathematics, Year 1
Additional Mathematics
Coordinate Geometry

7.1 Circles in Disguise

Consider the equation, $x^2 + y^2 + 12x - 2y = 27$

A friend of mine claims that this is a circle. If they are correct then it must be possible to algebraically manipulate this equation into the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where a, b and r are constants the values of which need to be found.
Then, the circle's centre would be (a, b) and its radius r.

7.2 Completing the Square

The technique employed is called "completing the square".

Teaching Video : http://www.NumberWonder.co.uk/v9033/7.mp4

[6 marks]

7.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 70

Question 1

Write each of the following in the “completed square” form,

$$y = (x + a)^2 + b$$

(i) $y = x^2 + 8x + 17$

(ii) $y = x^2 + 10x + 7$

(iii) $y = x^2 - 12x + 3$

(iv) $y = x^2 - 6x - 7$

[8 marks]

Question 2

Consider the circle, $x^2 - 4x + y^2 - 8y = 44$

(i) Rewrite this in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where *a*, *b* and *r* are constants the values of which are to be found.

[4 marks]

(ii) Hence, or otherwise, state;

(a) The coordinates of the centre of the circle

[1 mark]

(b) The radius of the circle.

[1 mark]

Question 3

Expand the brackets and simplify;

(i) $y = (x + 3)^2 + 4$

(ii) $y = (x + 7)^2 - 10$

(iii) $y = (x + 1)^2 + 7$

(iv) $y = \left(x + \frac{1}{2}\right)^2 + 10$

[8 marks]

Question 4

Consider the circle, $x^2 + y^2 + 8x - 14y + 29 = 0$

(i) Rewrite this in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where a, b and r are constants the values of which are to be found.

[4 marks]

(ii) Hence, or otherwise, state;

(a) The coordinates of the centre of the circle.

[1 mark]

(b) The radius of the circle.

[1 mark]

Question 5

Additional Mathematics Examination Question from June 2007, Q3 (OCR)

A circle has equation $x^2 + y^2 - 4x - 6y + 3 = 0$

Find the coordinates of the centre and the radius of the circle.

[4 marks]

Question 6

Additional Mathematics Examination Question from June 2015, Q9 (OCR)

The equation of a circle is $x^2 + y^2 - 8x + 2y - 19 = 0$

(i) Express the equation of C in the form $(x - a)^2 + (y - b)^2 = r^2$

[4 marks]

(ii) Hence or otherwise, use an algebraic method to decide whether the point (8, 3) lies inside, outside or on the circumference of the circle. Show all your working.

[2 marks]

Question 7

Additional Mathematics Examination Question from June 2010, Q9 (OCR)

The diameter of a circle is *PQ*, where *P* (1, 3) and *Q* (15, 1)

(i) Find the centre of the circle.

[2 marks]

(ii) Show that the radius of the circle is $5\sqrt{2}$

[2 marks]

(iii) Hence find the equation of the circle in the form,

$$x^2 + y^2 + ax + by + c = 0$$

[2 marks]

Question 8

Consider the circle $x^2 + y^2 + 41 = 10(x + y)$

(i) Rewrite this in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where *a*, *b* and *r* are constants the values of which are to be found.

[4 marks]

(ii) Hence, or otherwise, state;

(a) The coordinates of the centre of the circle.

[1 mark]

(b) The radius of the circle.

[1 mark]

Question 9

Additional Mathematics Examination Question from June 2005, Q12 (OCR)

(i) A circle has equation $x^2 + y^2 - 2x - 4y - 20 = 0$

Find the coordinates of its centre, C, and its radius.

[3 marks]

(ii) Find the coordinates of the points, A and B, where the line $y = x + 2$ cuts the circle

[5 marks]

(iii) Find angle ACB

[4 marks]

Question 10

A-Level Examination Question from May 2011, Paper C2, Q4 (Edexcel)

The circle C has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0$$

Find

(a) the coordinates of the centre of C,

[2 marks]

(b) the radius of C,

[2 marks]

(c) the coordinates of the points where C crosses the y-axis, giving your answers as simplified surds.

[4 marks]